MIXTURES AND ALLEGATIONS

MIXTURES

Mixing of two or more qualities of things produces a mixture. When two products of different qualities are mixed, the quality of the resultant mixture lies in between the qualities of the original constituent products, it will be higher than the lowest quality and lower than the highest quality of the items being mixed.

Ex. 17 kg of wheat Atta costing Rs. 29 per kg is mixed with 19 kg Rice Atta costing Rs. 25 per kg. Find the average price of the mixed Atta.

Sol. $\frac{(17\text{ kg} \times 29) + (19\text{ kg} \times 25)}{36} = Rs.\,26.88$ per kg.

If two types of a product of different prices per unit are mixed, the unit price of the resultant mixture will lie between the prices of the two types that form the mixture.

Here, the average quality is essentially the **weighted average** of the two constituent items.

If q_1 is the quantity or number of items of one particular item of quality P_2, and q_2 be the quantity or number of items of the second item of quality P_2 are mixed together to give a new mixture, then the **weighted average** value (p) of the quality of the mixture is given by

$$p = \frac{p_1 q_1 + p_2 q_2}{q_1 + q_2}$$

Ex. The Ratio of sugar and Salt in 45 litre solution is 5 : 4. How much salt is added to it to make the ratio 10 : 9.

Sol. Sugar = $\frac{45}{9} \times 5 = 25$

Salt = $\frac{45}{9} \times 4 = 20$

$\frac{25}{20+x} = \frac{10}{9}$

225 = 200 + x

x = 25

Salt quantity added in mixture = 25

If there are more than two groups of items mixed, the weighted average rule can be applied. We will only have to take figures (as shown in the formula for the two groups) for all the groups in the numerator as well as the denominator and calculate the weighted average. For example, if there are four groups of quantities q_1, q_2, q_3 and q_4 whose respective qualities are p_1, p_2, p_3 and p_4, then the weighted average quality of the group can be written as

$$p = \frac{p_1 q_1 + p_2 q_2 + p_3 q_3 + p_4 q_4}{q_1 + q_2 + q_3 + q_4}$$

A mixture can also be a solution - that is, a liquid mixed with another liquid which is normally water. The concentration of the solution is expressed as the proportion or percentage of the liquid in the total solution.

For example, if 20 litres of pure alcohol is mixed with 60 litres of water, then in a total solution of 80 litres, there is 20 litres of alcohol. So, concentration of this solution is 0.25 (= 20/80) or 25%.

Similarly, if 30 litres of pure milk is mixed with 10 litres of water, the concentration of this solution can be expressed as 75% (= 30/40) milk or 25% water.

We can also have two solutions mixed together to give a new solution. Such problems can also be handled in the same manner as other mixtures. In the weighted average rule, the quality of the constituents (p_1, p_2, etc.) will then be the concentrations of various solutions mixed together.

Ex. In an examination consisting of eight subjects Rohit scored 85, 85, 90, 91, 92 94, 95 and 96 marks. Find his average score.

Sol: The average score of Rohit

$$= \frac{\text{sum of the scores}}{\text{Number of subjects}}$$

$$= \frac{85 + 85 + 90 + 91 + 92 + 94 + 95 + 96}{8}$$

$$= \frac{728}{8} = 91 \text{ marks.}$$

Alternate Method:

The problem. can be solved using assumed average/mean also.

Let us take 90 as the assumed mean, then the actual average score

$$= \frac{-5 + -5 + 0 + 1 + 2 + 4 + 5 + 6}{8}$$

$$= 90 + \frac{8}{8} = 91 \text{ marks.}$$

ALLEGATIONS

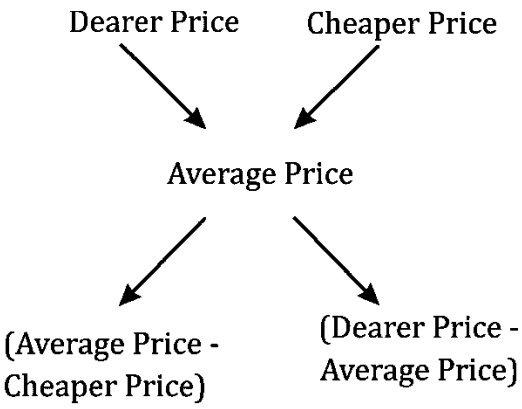
We will take the Weighted Average rule discussed in the previous section and rewrite the formula such that the quantity terms come on one side and the price terms come on the other side. If we do this, we get the rule $\frac{q_1}{q_2} = \frac{p - p_2}{p_1 - p}$

This is called the RULE OF ALLEGATION. This rule connects quantities and prices in mixtures. This can also be written as $\frac{q_1}{q_2} = \frac{p^2 - p}{p - p_1}$

In a descriptive manner, the Rule of Allegation can be written as

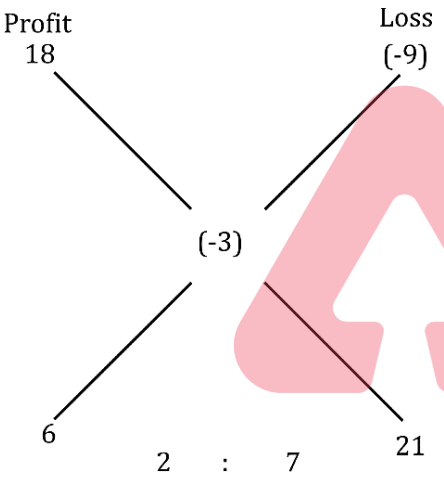
$$\frac{\text{Quantity of Cheaper}}{\text{Quantity of Dearer}} = \frac{\text{Rate of Dearer} - \text{Average Rate}}{\text{Average Rate} - \text{Rate of Cheaper}}$$

The above formula can be represented as follows:



Ex. A shopkeeper has 150kg of wheat, He sell one part of it at 18% profit and rest at 9% loss. On the whole sale his loss is 3%. Find the Quantity of Wheat sold at 9% loss?

Sol.



Quantity of Wheat sold at 9% loss = $\frac{150}{9} \times 7$

$= \frac{350}{3} = 116\frac{2}{3}\ kg$

If there is P volume of pure liquid initially and in each operation, Q volume is taken out and replaced by Q volume of water, then at the end of n such operations, the concentration (k) of the liquid in the solution is given

by $\left\{\frac{P-Q}{P}\right\}^n = k$

This gives the concentration (k) of the liquid as a PROPORTION of the total volume of the solution."
If the concentration has to be expressed as a percentage, then it will be equal to 100k.
If the volume of the liquid is to be found out at the end of n operations, it is given by KP. i.e., the concentration k multiplied by the total volume P of the solution.

Ex. An alloy contains Iron, Copper and Bronze in the ratio of 4 : 3 : 2. The quantity of Bronze that must be added to 99 kg of this alloy to have the new ratio 4 : 3 : 3 is

Sol.

	Iron	:	Copper	:	Bronze		
Old	4	:	3	:	2]	=	9
New	4	:	3	:	3] 1		

$9x = 99 \Longrightarrow x = 11$ kg

quantity of Bronze = 1 × 11 = 11kg