

6. MATHEMATICS

I. Real Analysis

Finite, countable and uncountable sets – Real Number system \mathbb{R} – infimum and supremum of a subset of \mathbb{R} – Bolzano – Weierstrass theorem.
 Sequences, convergence, limit superior and limit inferior of sequences, sub sequences, Heine Borel Theorem.
 Infinite series – Tests of convergence.
 Continuity and uniform continuity of real valued functions of real variable.
 Monotonic functions and functions of bounded variation.
 Differentiability and mean value theorems.
 Riemann integrability.
 Sequences and Series of functions.

II. Metric Spaces

Metric spaces – completeness, compactness and connectedness – continuity and uniform continuity of functions from one metric space into another.
 Topological spaces – base and subbase – continuous function.

III. Elementary Number

Primes and composite numbers – Fundamental Theorem of arithmetic – divisibility congruences – Fermat's theorem – Wilson's Theorem – Euler's ϕ - function.

IV. Group Theory

Groups, subgroups, normal subgroups – quotient groups – homomorphisms and isomorphism theorems – permutation groups, cyclic groups, Cayley's theorem. Sylow's theorems and their applications.

V. Ring Theory

Rings, integral domains, fields – subrings and ideals – Quotient rings – homomorphisms – Prime ideals and maximal ideals – polynomial rings – Irreducibility of polynomials – Euclidean domains and principal ideal domains.

VI. Vector Spaces

Vector Spaces, Subspaces – Linear dependence and independence of vectors – basis and dimension – Quotient spaces – Inner product spaces – Orthonormal basis – Gram – Schmidt process.

VII. Matrix Theory

Linear transformations – Rank and nullity – change of bases.
 Matrix of a linear transformation – singular and non-singular matrices – Inverse of matrix – Eigenvalues and eigenvectors of matrix and of linear transformation – Cayley Hamilton's theorem.

VIII. Complex Analysis

Algebra of complex numbers – the complex plane – Complex functions and their Analyticity – Cauchy-Riemann equations – Mobius transformations.
 Power Series.
 Complex Integration – Cauchy's theorem – Morera's Theorem – Cauchy's integral formula – Liouville's theorem – Maximum modulus principle – Schwarz's lemma – Taylor's series – Laurents series.
 Calculus of residues and evaluation of integrals.

IX. Ordinary Differential Equation

Ordinary Differential Equation (ODE) of first order and first degree – Different methods of solving them – Exact Differential equations and integrating factors.
 ODE of first order and higher degree – equations solvable for p , x and y – Clairaut's equations – Singular Solutions.
 Linear differential equations with constant coefficients and variable coefficients – variation of parameters.

X. Partial Differential Equations

Formation of differential equations (PDE) – Lagrange and Charpit methods for solving first order – PDE's – Cauchy problem for first order PDE's Classification of second order PDE's – General solution of higher order PDE's with constant coefficients.