Test Booklet Code & Serial No. **A**
प्रश्नपत्रिका कोड व क्रमांक

Paper-II
MATHEMATICAL SCIENCE

Signature and Name of Invigilator

1. (Signature) ....
   (Name) ....
2. (Signature) ....
   (Name) ....

Seat No. | | | | | | | |
(In figures as in Admit Card)

Seat No. ....
(In words)

OMR Sheet No. | | | | | | | |
(To be filled by the Candidate)

JUN - 30225

Time Allowed : 2 Hours] **[Maximum Marks : 200**

Number of Pages in this Booklet : **48** Number of Questions in this Booklet : **180**

Instructions for the Candidates

1. Write your Seat No. and OMR Sheet No. in the space provided on the top of this page.
2. This paper consists of **180** objective type questions. Each question will carry *two* marks. Candidates should attempt *all* questions either from sections I & II or from sections I & III only.
3. At the commencement of examination, the question booklet will be given to the student. In the first 5 minutes, you are requested to open the booklet and compulsorily examine it as follows :
   (*i*) To have access to the Question Booklet, tear off the paper seal on the edge of this cover page. Do not accept a booklet without sticker-seal or open booklet.
   (*ii*) Tally the number of pages and number of questions in the booklet with the information printed on the cover page. Faulty booklets due to missing pages/questions or questions repeated or not in serial order or any other discrepancy should not be accepted and correct booklet should be obtained from the invigilator within the period of 5 minutes. Afterwards, neither the Question Booklet will be replaced nor any extra time will be given. The same may please be noted.
   (*iii*) After this verification is over, the OMR Sheet Number should be entered on this Test Booklet.
4. Each question has four alternative responses marked (A), (B), (C) and (D). You have to darken the circle as indicated below on the correct response against each item.
   Example : where (B) is the correct response.
   (A) ● (C) (D)
   Following wrong methods should not be used as they are not recognised by scanning machine in digitized assessment. Candidate using such method will be responsible for their loss.
   WRONG METHODS
   (A)(B)(C)(D) (A)(B)(C)(D) (A)(B)(C)(D) (A)(B)(C)(D)
5. Your responses to the items are to be indicated in the **OMR Sheet given inside the Booklet only.** If you mark at any place other than in the circle in the OMR Sheet, it will not be evaluated.
6. Read instructions given inside carefully.
7. Rough Work is to be done at the end of this booklet.
8. If you write your Name, Seat Number, Phone Number or put any mark on any part of the OMR Sheet, except for the space allotted for the relevant entries, which may disclose your identity, or use abusive language or employ any other unfair means, you will render yourself liable to disqualification.
9. You have to return original OMR Sheet to the invigilator at the end of the examination compulsorily and must not carry it with you outside the Examination Hall. You are, however, allowed to carry the Test Booklet and duplicate copy of OMR Sheet on conclusion of examination.
10. Use only Blue/Black Ball point pen.
11. Use of any calculator or log table, etc., is prohibited.
12. There is no negative marking for incorrect answers.

विद्यार्थ्यांसाठी महत्त्वाच्या सूचना

1. परीक्षार्थींनी आपला आसन क्रमांक या पृष्ठावरील वरच्या कोपऱ्यात लिहावा. तसेच आपणांस दिलेल्या उत्तरपत्रिकेचा क्रमांक त्याखाली लिहावा.
2. सदर प्रश्नपत्रिकेत 180 बहुपर्यायी प्रश्न आहेत. प्रत्येक प्रश्नास **दोन** गुण आहेत. विद्यार्थ्यांनी खण्ड I व II किंवा खण्ड I व III मधील **सर्व** प्रश्न सोडविणे अनिवार्य आहे.
3. परीक्षा सुरू झाल्यावर विद्यार्थ्याला प्रश्नपत्रिका दिली जाईल. सुरुवातीच्या 5 मिनिटांमध्ये आपण सदर प्रश्नपत्रिका उघडून खालील बाबी अवश्य तपासून पहाव्यात.
   (*i*) प्रश्नपत्रिका उघडण्यासाठी प्रश्नपत्रिकेवर लावलेले सील उघडावे. सील नसलेली किंवा सील उघडलेली प्रश्नपत्रिका स्वीकारू नये.
   (*ii*) पहिल्या पृष्ठावर नमूद केल्याप्रमाणे प्रश्नपत्रिकेची एकूण पृष्ठे तसेच प्रश्नपत्रिकेतील एकूण प्रश्नांची संख्या पडताळून पहावी. पृष्ठे कमी असलेली/कमी प्रश्न असलेली/प्रश्नांचा चुकीचा क्रम असलेली किंवा इतर त्रुटी असलेली सदोष प्रश्नपत्रिका सुरुवातीच्या 5 मिनिटातच पर्यवेक्षकाला परत देऊन दुसरी प्रश्नपत्रिका मागवून घ्यावी. त्यानंतर प्रश्नपत्रिका बदलून मिळणार नाही तसेच वेळही वाढवून मिळणार नाही याची कृपया विद्यार्थ्यांनी नोंद घ्यावी.
   (*iii*) वरीलप्रमाणे सर्व पडताळून पाहिल्यानंतरच प्रश्नपत्रिकेवर ओ.एम.आर. उत्तरपत्रिकेचा नंबर लिहावा.
4. प्रत्येक प्रश्नासाठी (A), (B), (C) आणि (D) अशी चार विकल्प उत्तरे दिली आहेत. त्यातील योग्य उत्तराचा रकाना खाली दर्शविल्याप्रमाणे ठळकपणे काळा/निळा करावा.
   उदा. : जर (B) हे योग्य उत्तर असेल तर.
   (A) ● (C) (D)
   खालील चुकीच्या पद्धती वापरू नये, कारण डिजिटाइज्ड (Digitized) मूल्यांकनात स्कॅनिंग मशीन त्यांना ओळखत नाही. त्या पद्धती वापरून नुकसान झाल्यास त्यास विद्यार्थीच जबाबदार असतील.
   WRONG METHODS
   (A)(B)(C)(D) (A)(B)(C)(D) (A)(B)(C)(D) (A)(B)(C)(D)
5. या प्रश्नपत्रिकेतील प्रश्नांची उत्तरे **ओ.एम.आर. उत्तरपत्रिकेतच दर्शवावीत.** इतर ठिकाणी लिहिलेली उत्तरे तपासली जाणार नाहीत.
6. आत दिलेल्या सूचना काळजीपूर्वक वाचाव्यात.
7. प्रश्नपत्रिकेच्या शेवटी जोडलेल्या कोऱ्या पानावरच कच्चे काम करावे.
8. जर आपण ओ.एम.आर. वर नमूद केलेल्या ठिकाणाव्यतिरिक्त इतर कोठेही नाव, आसन क्रमांक, फोन नंबर किंवा ओळख पटेल अशी कोणतीही खूण केलेली आढळून आल्यास अथवा असभ्य भाषेचा वापर किंवा इतर गैरमार्गांचा अवलंब केल्यास विद्यार्थ्याला परीक्षेस अपात्र ठरविण्यात येईल.
9. परीक्षा संपल्यानंतर विद्यार्थ्याने मूळ ओ.एम.आर. उत्तरपत्रिका पर्यवेक्षकांकडे परत करणे आवश्यक आहे. तथापि, प्रश्नपत्रिका व ओ.एम.आर. उत्तरपत्रिकेची द्वितीय प्रत आपल्याबरोबर नेण्यास विद्यार्थ्यांना परवानगी आहे.
10. फक्त निळ्या किंवा काळ्या बॉल पेनचाच वापर करावा.
11. कॅलक्युलेटर किंवा लॉग टेबल वापरण्यास परवानगी नाही.
12. चुकीच्या उत्तरासाठी गुण कपात केली जाणार नाही.

Mathematical Science
Paper II

Time Allowed : 120 Minutes] **[Maximum Marks : 200**

Note : This paper contains **One Hundred Eighty (180)** multiple choice questions in THREE (3) sections, each question carrying **TWO (2)** marks. Attempt ***all*** questions either from **Sections I & II** only **or from Sections I & III** only. The OMR sheets with questions attempted from both the Sections viz. **II & III, will not be assessed.**

Number of questions, sectionwise :

Section I : Q. Nos. 1 to 20, **Section II : Q. Nos. 21 to 100,** **Section III : Q. Nos. 101 to 180.**

SECTION I

1. Let X be a connected metric space. Then which of the following is true ?

(A) X is finite

(B) X is infinite

(C) $|X| \neq 10$

(D) X is uncountable

2. Let X, Y be metric spaces. Suppose $f, g : X \to Y$ are continuous functions and $D = \{x \mid f(x) = g(x)\}$. Then $f \equiv g$ if D is :

(A) Dense in X

(B) Unbounded

(C) Compact

(D) Closed

3. Which of the following statements is *not* true in any metric space ?

(A) Every open set is closed

(B) Every subset is bounded

(C) There is a finite set which is not closed

(D) There is a countable set which is not compact

4. The series :

$$\frac{2}{1} + \frac{3}{4} + \frac{4}{9} + \ldots\ldots + \frac{n+1}{n^2} + \ldots\ldots$$

(A) Converges to an irrational number

(B) Converges to a positive transcendental number

(C) Converges to a rational number

(D) Diverges

5. For any two sequences $\{x_n\}$ and $\{y_n\}$, it is known that,

$$\underline{\lim}\ x_n + \underline{\lim}\ y_n \leq \underline{\lim}\ (x_n + y_n) \leq$$

$$\overline{\lim}\ (x_n + y_n) \leq \overline{\lim}\ x_n + \overline{\lim}\ y_n \quad (*)$$

Suppose that $x_n = \begin{cases} 0 & \text{if } n \text{ is even} \\ 1 & \text{if } n \text{ is odd} \end{cases}$

and $y_n = 1 - x_n$, for all $n \geq 1$. Then, which of the following statements is/are true for $\{x_n\}$ and $\{y_n\}$?

S_1 : strict inequalities hold in (*) above.

S_2 : $\{x_n + y_n\}$ converges, while $\{x_n\}$ and $\{y_n\}$ do not.

(A) Neither S_1 nor S_2 is true

(B) Both S_1 and S_2 are true

(C) Only S_1 is true

(D) Only S_2 is true

6. Suppose that $\{f_n\}$ is a sequence of functions defined by :

$$f_n(x) = \frac{x^n}{1 + x^n}, \ 0 \leq x \leq 1.$$

Then, which of the following statements is true ?

S_1 : $f_n \to f \equiv 0$ uniformly on [0, 1]

S_2 : $\sup\limits_{0 \leq x \leq 1/2} f_n(x) \to F(x) \equiv 0$ on [0, ½]

S_3 : $f_n \to f = 0$ uniformly on [0, ½]

(A) Only S_1 and S_2 are true

(B) Only S_1 and S_3 are true

(C) Only S_2 and S_3 are true

(D) All the three are true

7. Suppose that f is a function defined by :

$$f(x) = \frac{x}{3 - 2x}, \quad 0 < x < 1$$

If $g(x) = f''(x) + f'(x) - [f'(x)]^2$, then $g(1)$ is equal to :

(A) 9

(B) 2

(C) 6

(D) 4

8. Suppose that f is a function on $\mathbf{R}^2$ defined by :

$$f(x, y) = \begin{cases} \dfrac{xy}{x^2 + y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$$

Then, which of the following statements is/are true ?

S_1 : f is continuous at (0, 0).

S_2 : Both partial derivatives exist at (0, 0).

(A) Both S_1 and S_2 are true

(B) Only S_1 is true

(C) Only S_2 is true

(D) Neither S_1 nor S_2 is true

9. Consider the function f defined by :

$$f(x) = \begin{cases} \dfrac{\lambda \cos(x)}{\pi/2 - x}, & x \neq \pi/2 \\ 1, & x = \pi/2 \end{cases}$$

What should be the value of λ so that f would be continuous at $x = \pi/2$?

(A) 1

(B) –1

(C) 2

(D) –2

10. The number of non-empty proper subsets of X = {1, 2, 3, 4, 5} is :

(A) 16

(B) 31

(C) 32

(D) 30

11. Which of the following statements is true ?

(A) $\mathbf{C}$ is a vector space over $\mathbf{R}$ of dimension ∞

(B) $\mathbf{R}$ is a vector space over $\mathbf{Q}$ of dimension ∞

(C) The vector space consisting of all polynomials with real coefficients of degree upto n has dimension n

(D) Every set of vectors in a vector space V can be extended to a basis of V

12. Let $T : \mathbf{R}^3 \to \mathbf{R}^2$ be a linear map such that :

$$T\begin{bmatrix}1\\0\\0\end{bmatrix} = \begin{bmatrix}1\\0\end{bmatrix}, \quad T\begin{bmatrix}0\\1\\0\end{bmatrix} = \begin{bmatrix}0\\1\end{bmatrix} \text{ and}$$

$$T\begin{bmatrix}0\\1\\1\end{bmatrix} = \begin{bmatrix}1\\1\end{bmatrix}.$$

Then $T\begin{bmatrix}1\\1\\1\end{bmatrix} =$

(A) $\begin{bmatrix}2\\1\end{bmatrix}$

(B) $\begin{bmatrix}1\\1\end{bmatrix}$

(C) $\begin{bmatrix}-1\\2\end{bmatrix}$

(D) $\begin{bmatrix}2\\2\end{bmatrix}$

13. Assuming $(X - I)^{-1}$ exists, what will be $I + X + X^2 + \ldots\ldots + X^{n-1}$?

(A) $(I - X)^n (X - I)^{-1}$

(B) $(I - X^n) (X - I)^{-1}$

(C) $(X^n - I) (X - I)^{-1}$

(D) $(X - I)^{-1} (X - I)^n$

14. Suppose A, B and C are 3×3 matrices with $|A| = 3$ and $|B| = 3$. Then, $|2\ C\ A^2\ B^T\ (CA)^{-1}\ B|$ is :

(A) 54

(B) 216

(C) 27

(D) 172

15. The following system of equations :

$$\begin{aligned} x + z &= 3 \\ x - y - z &= 1 \\ -x + y &= 1 \end{aligned}$$

(A) is consistent with infinitely many solutions

(B) is consistent with a unique solution

(C) is not consistent

(D) has exactly three solutions

16. Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 2 & 0 \\ 1 & 1 & 4 \end{bmatrix}$. Then, the eigenvalues of the matrix $A^2 - 3A + 2I$ are :
    (A) 0, 0, 6
    (B) –1, 1, 0
    (C) –2, 4, 18
    (D) 0, 3, 12

17. Suppose A is a non-zero real nilpotent matrix. Then :
    (A) A is not triangulable
    (B) A has at least two eigenvalues
    (C) The transpose of A is nilpotent
    (D) The minimal polynomial of A is same as its characteristic polynomial

18. Let A be an $n \times n$ matrix with all entries equal to 1. Then :
    (A) All eigenvalues of A are positive
    (B) 0 is an eigenvalue of A with multiplicity 1
    (C) A has at least three distinct eigenvalues
    (D) A has exactly one non-zero eigenvalue

19. Let A be a 5×5 real matrix and let B be the adjoint of A. Then :
    (A) A and B have same rank
    (B) A and B have same determinant
    (C) A is invertible if and only if B is invertible
    (D) $AB = 5 I_5$

20. The quadratic form corresponding to the matrix

$$\begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

is :
    (A) Positive definite
    (B) Negative definite
    (C) Positive semidefinite
    (D) Negative semidefinite

SECTION II

21. For a complex number z, $|e^{-iz}| > 1$ holds if :

(A) Re $z > 0$

(B) Re $z < 0$

(C) Im $z > 0$

(D) Im $z < 0$

22. Under the stereographic projection between S^2 and $\mathbf{C}_\infty$, the point $1 + i$ of $\mathbf{C}$ corresponds to the following point of S^2.

(A) $\left(\frac{2}{3}, \frac{-2}{3}, \frac{1}{3}\right)$

(B) $\left(\frac{2}{3}, \frac{2}{3}, \frac{1}{3}\right)$

(C) $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right)$

(D) $\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$

23. Which of the following is true ?

(A) There exists an analytic function f such that Re $f(z) = y^2 - 2x$

(B) There exists an analytic function f such that Im $f(z) = x^2 - y$

(C) If f is analytic on the unit disk Δ, then the function $\overline{f}(\overline{z})$ is analytic on Δ

(D) There exists a real-valued non-constant function which is analytic on a domain D

24. The function $f(z) = \dfrac{\sin z}{1 - e^z}$ has :

(A) a removable singularity at $z = 0$

(B) a simple pole at $z = 0$

(C) a pole of order two at $z = 0$

(D) an essential singularity at $z = 0$

25. Let Δ denote the unit disk and $f : \Delta \to \overline{\Delta}$ be analytic with $f(0) = 0$. Then :

(A) If $f\left(\frac{1}{2}\right) = \frac{1}{2}$, then $f(z) = z$ for all $z \in \Delta$

(B) $|f'(0)| = 1$, then $f(z) = z$ for all $z \in \Delta$

(C) $|f(z)| \geq |z|$ for all $z \in \Delta$

(D) If $f'(0) = 0$, then f is constant

26. If $\gamma = \{z \in \mathbf{C} / |z - \frac{\pi}{2}| = 1\}$, then the value of $\int_{\gamma} \frac{\sin z \, dz}{\left(z - \frac{\pi}{2}\right) z}$ is :

(A) 0

(B) $2\pi i$

(C) $4i$

(D) $-2\pi i$

27. Which of the following Möbius transformations maps the unit disk onto the right half plane $\{w / \text{Re } w > 0\}$?

(A) $f(z) = \frac{z-i}{z+i}$

(B) $f(z) = \frac{z-1}{z+1}$

(C) $f(z) = \frac{z+1}{2z-1}$

(D) $f(z) = \frac{z+1}{1-z}$

28. Which of the following power series does not have radius of convergence 1 ?

(A) $1 - z + z^2 - z^3 + ...$

(B) $z - \frac{z^2}{2} + \frac{z^3}{3} - \frac{z^4}{4} + ...$

(C) $2 - 2.3z + 3.4z^2 - 4.5z^3 + ...$

(D) $\frac{1}{2} + \frac{z}{2^2} + \frac{z^2}{2^3} + ...$

29. An analytic function f on unit disk Δ is constant on Δ if :

(A) f is bounded

(B) f^2 is analytic

(C) $\overline{f} + f$ is analytic

(D) $\frac{1}{f}$ is analytic

30. Which of the following functions is conformal on the unit disk ?

(A) $z^5 + 1$

(B) $\cos z$

(C) ze^{z^3+1}

(D) ze^z

31. Consider the following two statements :

(*i*) An entire function with infinitely many zeros is identically zero.

(*ii*) There exists an analytic function f on the unit disk such that :

$$f\left(\frac{1}{2n}\right) = \frac{1}{n} = f\left(\frac{1}{2n+1}\right)$$

for $n = 1, 2, 3 .....$

Then :

(A) Only (*i*) is true

(B) Only (*ii*) is true

(C) Both (*i*) and (*ii*) are true

(D) Both (*i*) and (*ii*) are false

32. Suppose f is analytic in a disk D and $f(z) \neq 0$ for all $z \in \mathrm{D}$. Then :

(A) f is a one-one map

(B) f is bounded

(C) f' need not be analytic

(D) there is an analytic function $g : \mathrm{D} \to \mathbf{C}$ such that $f(z) = e^{g(z)}$

33. Let $f(z) = \sin \frac{1}{z-1}$ and $\mathrm{A} = \{z / 0 < |z-1| < 1\}$ Then :

(A) f(A) is a bounded set

(B) f(A) is contained in a half-plane

(C) $\overline{f(\mathrm{A})} = \mathbf{C}$

(D) f is not analytic on A

34. One of the values of $\sqrt{1+i}$, where $i = \sqrt{-1}$, is :

(A) $e^{\frac{1}{4}\left(\log 2 + \frac{\pi}{2} i\right)}$

(B) $e^{\frac{1}{4}\left(\log 2 + \frac{\pi}{4} i\right)}$

(C) $e^{\frac{1}{4}(\log 2 + \pi i)}$

(D) $e^{\frac{1}{2}\left(\log 2 + \frac{\pi}{2} i\right)}$

35. What is the minimal polynomial of $\sqrt{2} + \sqrt{3}$ over $\mathbf{Q}$?

(A) $x^3 - 5x^2 + 1$

(B) $x^4 - 6x^2 + 1$

(C) $x^4 - 10x^2 + 1$

(D) $x^2 + 6x + 1$

36. What is the Galois group of $f(x) = x^3 - 3x^2 + 15x - 3$ over $\mathbf{Q}$?
(A) $(\mathbf{Z}_3, +)$
(B) $\mathbf{Z}_4, +)$
(C) the alternating group A_4
(D) the permutation group S_3

37. There is no subfield with k elements to the field with 64 elements if k is :
(A) 16
(B) 4
(C) 8
(D) 2

38. Let $\mathbf{F}_q$ be a finite field with q elements. Then which of the following need *not* be true ?
(A) q is a power of a prime
(B) For every positive integer n, there is an irreducible polynomial of degree n in $\mathbf{F}_q[x]$.
(C) The multiplicative group $(\mathbf{F}_q \setminus \{0\}, X)$ is cyclic
(D) $\mathbf{F}_q$ is algebraically closed

39. For which of the following n, the polynomial $x^3 - 9$ is irreducible in $\mathbf{Z}_n\ [x]$?
(A) 11
(B) 31
(C) 9
(D) 10

40. Consider the following statements :
I : A subring of a unique factorization domain is a unique factorization domain.
II : Let R be a unique factorization domain and $d = \gcd(a, b)$ for $a, b \in R$. Then $(d) = (a) + (b)$.
Which is correct ?
(A) Only I is true
(B) Only II is true
(C) Both I and II are true
(D) Neither I nor II is true

41. Consider the following statements :
I : Any finite non-commutative ring must have at least 16 elements.
II : Let R be a ring with unity such that $|R| = 9$. Then R is commutative.
Which of the following is true ?
(A) Only I is true
(B) Only II is true
(C) Both I and II are true
(D) Neither I nor II is true

42. The special linear group $SL_2(\mathbf{Z}/2\mathbf{Z})$ of 2×2 matrices over $\mathbf{Z}/2\mathbf{Z}$:
    - (A) has cardinality 8
    - (B) has cardinality 16
    - (C) is isomorphic to S_3
    - (D) is isomorphic to A_4

43. Consider the following statements :

    I : If G is a group such that every cyclic group of G is a normal subgroup of G, then every subgroup of G is a normal subgroup of G.

    II : If H is a subgroup of a group G such that the index of H in G is 2, then H is a normal subgroup of G.

    Which of the following statement(s) is/are true ?
    - (A) Only I is true
    - (B) Only II is true
    - (C) Both I and II are true
    - (D) Neither I nor II is true

44. Which of the folowing is correct ?
    - (A) A_4 has a subgroup of order 6
    - (B) A_4 has only one proper non-trivial normal subgroup
    - (C) A_4 is simple
    - (D) Two elements of A_n that are conjugates in S_n are always conjugates in A_n also

45. Let G be a group of order p^2, (p-prime). Then which of the following is correct ?
    - (A) G has exactly 3 subgroups
    - (B) G has exactly 2 subgroups
    - (C) G has exactly 2 non-trivial, proper subgroups
    - (D) G has more than 4 subgroups

46. Suppose given a positive integer k has the property that for any integer $n \geq k$ can be written as a sum of 3's and/or 8's. Then which of the following is the value of k ?
    - (A) 6
    - (B) 8
    - (C) 9
    - (D) 15

47. How many units are there in $\mathbf{Z}_{12}$?

(A) 3

(B) 4

(C) 5

(D) 2

48. The permutation (12) (13) in S_4 is similar to :

(A) (2 3 4)

(B) (2 3)

(C) (1 2 3 4)

(D) (1 2) (3 4)

49. Let B be the collection of bounded real sequences with sup norm $\| \bullet \|_\infty$. Let

$A_0 = \{\{x_n\} / x_n = 0 \ \forall \ n \geq 10\}$ and

$B_0 = \{\{x_n\} / \| \{x_n\} \|_\infty \leq 1\}$.

Then :

(A) A_0 is compact

(B) B_0 is compact

(C) A_0 is not closed

(D) $A_0 \cup B_0$ is compact

50. The interval (–1, 1) is homeomorphic to :

(A) The parabola $y = x^2$

(B) The ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$

(C) The circle $x^2 + y^2 = 1$

(D) The interval [–1, 1]

51. Let C[0, 1] denote the space of continuous functions on [0, 1] with sup norm $\| \ \|_\infty$. The set

$M = \{f \in C\{0, 1\} / f\left(\frac{1}{2}\right) \neq 0\}$ is :

(A) Bounded subset of C[0, 1]

(B) Open subset of C[0, 1]

(C) Closed subset of C[0, 1]

(D) Compact subset of C[0, 1]

52. Let $f : X \to \{+1, -1\}$ be a non-constant map defined on a topological space X. Then :

(A) f is never continuous

(B) f is continuous iff X is disconnected

(C) f is continuous iff X is connected

(D) f is continuous iff X is compact

53. Which of the following statements is false for a space X with discrete topology ?

(A) Every open set in X is closed

(B) Every closed set is compact in X

(C) Every closed set is open in X

(D) Every singleton set is open in X

54. Let C be a closed subset of **R** such that $C \cap I \neq \phi$ for any open interval I of **R**. Then :

(A) C is compact

(B) C is bounded

(C) C = **R**

(D) C is countable

55. Let $S^{'}$ denote the unit circle with centre origin which of the following space is not connected ?

(A) $S' \times [0, 1]$

(B) $S' \setminus \{P\} \times [0, 1]$ where P is a given fixed point on S'

(C) $S' \setminus \{P_1, P_2\} \times [0, 1]$ where $P_1, P_2 \in S'$ are distinct points

(D) $S' \times \{0\}$

56. A topological space X is normal if :

(A) X is T_2

(B) X is regular and T_2

(C) for given $x \in X$ and a neighbourhood u of x there exists an open set v such that $x \in \bar{V} \subset u$

(D) for a given closed set $A \subset X$ and an open set u containing A, there is an open set V such that $A \subset \bar{V} \subset u$

57. Let $X = \{(x, y) \in \mathbf{R}^2 : x^2 y^2 = 1\}$. Then :

(A) X is compact and connected

(B) X is compact, but not connected

(C) X is connected, but not compact

(D) X is neither compact nor connected

58. Let τ_1 and τ_2 be the topologies induced by the metrics d_1 and d_2 on a set X such that $d_1 \leq d_2$. For $x \in X$ and $r > 0$, the open balls in (X, d_1) and (X, d_2) are denoted by $B_1(x, r)$ and $B_2(x, r)$ respectively. Then :

(A) $B_1(x, r) \subset B_2(x, r)$ and $\tau_1 \subset \tau_2$

(B) $B_1(x, r) \subset B_2(x, r)$ and $\tau_2 \subset \tau_1$

(C) $B_2(x, r) \subset B_1(x, r)$ and $\tau_1 \subset \tau_2$

(D) $B_2(x, r) \subset B_1(x, r)$ and $\tau_2 \subset \tau_1$

59. Let τ be the topology generated by $\{(a, b] \mid a, b \in \mathbf{R}, a < b\}$, on $\mathbf{R}$. Let $a_n = \frac{1}{n}$ and $b_n = -\frac{1}{n}$ for all $n \in \mathbf{N}$. Then (the sequences) :

(A) (a_n) and (b_n) converges in τ

(B) (a_n) converges and (b_n) does not converge in τ

(C) (a_n) does not converge and (b_n) converges in τ

(D) neither (a_n) nor (b_n) converges in τ

60. Let $X = \bigcap_{n=1}^{\infty} I_n$, where $I_1 \underset{\neq}{\supset} I_2 \underset{\neq}{\supset} I_3 \ldots\ldots \underset{\neq}{\supset} I_n \underset{\neq}{\supset} \ldots\ldots$ and every I_n is a non-empty closed and bounded interval in $\mathbf{R}$. Then :

(A) $X = \phi$

(B) $|X| = 1$

(C) $|X| = \infty$

(D) $|X| > 0$

61. The differential equation corresponding to the family of curves :

$$y = c(x - c)^2,$$

where c is an arbitrary constant, is :

(A) $(y')^2 = 2y(x^2y' + 3y)$

(B) $(y')^3 = 4y(xy' - 2y)$

(C) $(y')^4 = 3x(x^3y' + 4y)$

(D) $(y')^2 = 3y(xy' + 2y)$

62. Let ϕ and ψ be solution of

$$y'' - 2xy' + (\sin x^2)y = 0,$$

such that :

$$\phi(0) = 1, \ \phi'(0) = 1$$
$$\psi(0) = 1, \ \psi'(0) = 2$$

Then W (ϕ, ψ) at x = 1 is :

(A) $e - 1$

(B) $e^2 + 1$

(C) $1/e$

(D) $e^{1/2}$

63. For the differential equation :

$$y' = x \sin y + y \cos x$$

defined on rectangle

$$|x| \leq 5, \ |y| \leq 2,$$

which of the following is Lipschitz constant ?

(A) 4

(B) 6

(C) 1

(D) 3

64. Consider the following statements for any solution ϕ of the differential equation with constant coefficient :

$$y'' + a_1 y' + a_2 y = 0,$$

where a_1, a_2 are constants.

(I) $\phi \to 0$ as $x \to \infty$, if the real parts of the roots of the characteristic polynomial are negative.

(II) $\phi \to 0$ as $x \to \infty$, if the real parts of the roots of the characteristic polynomial are positive.

Then :

(A) Only I is true

(B) Only II is true

(C) Both I and II are true

(D) Both I and II are false

65. Consider the differential equation

$$\frac{dy}{dx} - y + y^2 = 0.$$

Then $\lim_{x \to \infty} y(x)$ equals.

(A) 0

(B) 1

(C) –1

(D) ∞

66. The general solution of

$$\frac{dy}{dx} + 2xy = 2e^{-x^2}$$

is :

(A) $y = (2x + c)e^{-x^2}$

(B) $y = 2ce^{-x^2}$

(C) $y = ce^{\frac{-x^2}{2}}$

(D) $y = (2x^2 + c)\, e^{-x^2}$

67. The solutions of the differential equations

$$\frac{dy}{dx} = 2y + z \quad \text{and} \quad \frac{dz}{dx} = 3y$$

satisfy the relation :

(A) $y - z = ce^{3x}$

(B) $y + z = ce^{-x}$

(C) $3y + z = ce^{3x}$

(D) $3y + z = ce^{-x}$

68. The normals to the surfaces represented by the partial differential equations :

$$yzdx + xzdy + xydz = 0 \quad \text{and}$$

$$yzp + xzq = xy$$

(A) are parallel

(B) intersect

(C) intersect orthogonally

(D) are coincide

69. The partial differential equation

$$yp - xq = 0$$

has :

(A) unique integral surface containing the circle $x^2 + y^2 = 4$, $z = 2$

(B) finitely many integral surfaces containing the circle $x^2 + y^2 = 4$, $z = 2$

(C) infinitely many surfaces containing the circle $x^2 + y^2 = 4$, $z = 2$

(D) no integral surface containing the circle $x^2 + y^2 = 4$, $z = 2$

70. A partial differential equation can have :

(I) unique general integral

(II) unique complete integral

(III) unique integral surface containing the given curve.

Then :

(A) (I), (II) and (III) are true

(B) (I), (II) and (III) are false

(C) (II) and (III) are true

(D) (I) and (II) are true

71. Consider the following two statements :

(I) The characteristic curve of a one parameter family of surfaces $f(x, y, z, a) = 0$ is a locus of the common points of intersection of :

$$f(x, y, z, a) = 0 \text{ and}$$

$$\frac{\partial f(x, y, z, a)}{\partial a} = 0.$$

(II) The envelope of the one parameter family of surfaces $f(x, y, z, a) = 0$ is obtained by eliminating the parameter 'a' from the equations :

$$f(x, y, z, a) = 0 \text{ and}$$

$$\frac{\partial f(x, y, z, a)}{\partial a} = 0.$$

Then :

(A) (I) is true and (II) is false

(B) (I) is false and (II) is true

(C) (I) and (II) both are false

(D) (I) and (II) both are true

72. Let $P(x, y, z)dx + Q(x, y, z)dy + R(x, y, z)\,dz = 0$ be a Pfaffian differential equation.

Consider the following three statements.

(I) Every Pfaffian differential equation is integrable.

(II) A Pfaffian differential equation is integrable if and only if $\bar{X} \cdot \text{curl } \bar{X} = 0$, where $\bar{X} = (P, Q, R)$.

(III) A Pfaffian differential equation is integrable if and only if :

$$P\left(\frac{\partial Q}{\partial z} - \frac{\partial R}{\partial y}\right) + Q\left(\frac{\partial R}{\partial x} - \frac{\partial P}{\partial z}\right) + R\left(\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x}\right) = 0$$

Then :

(A) (I) and (II) are true

(B) (I) and (III) are true

(C) (II) and (III) are true

(D) (I), (II) and (III) are true

73. The region in which the partial differential equation,

$$yu_{xx} + 2xyu_{xy} + xu_{yy} = u_x + u_y$$

is hyperbolic if :

(A) $xy \neq 1$

(B) $xy \neq 0$

(C) $xy < 0$

(D) $xy < 1$

74. The envelope of the one parameter family of planes through (0, 0, 0) to the integral surface of the partial differential equation $p^2 + q^2 = 1$ is :

(A) $x^2 - y^2 = 4$

(B) $x^2 + y^2 = 4$

(C) $x^2 + y^2 = z^2$

(D) $x^2 - z^2 = y^2$

75. Consider the following statements :

(I) Wave equation is of parabolic type.

(II) Heat equation is of elliptic type.

Then :

(A) Only (I) is true

(B) Only (II) is true

(C) Both (I) and (II) are true

(D) Both (I) and (II) are false

76. Let Δ be the forward difference operator, ∇ be the backward difference operator and δ be the central difference operator. Then :
(A) $\nabla \Delta = \Delta - \nabla$
(B) $\nabla \Delta \neq \delta^2$
(C) $\Delta - \nabla \neq \delta^2$
(D) $\nabla \Delta \neq \Delta - \nabla$

77. Let $f(x)$ be a polynomial of degree n. Then the nth forward difference of $f(x)$, that is, $\Delta^n [f(x)]$ is :
(A) always 0
(B) constant
(C) always 1
(D) cannot determine

78. $\blacktriangle$ is a divided difference operator and Δ is a forward difference operator, $f(x)$ and $g(x)$ are two functions and a and b are constants, then :
(A) $\blacktriangle [a f(x) + bg(x)] = a \blacktriangle f(x) + b \blacktriangle g(x)$
(B) $\blacktriangle [af(x) + bg(x)] = a \blacktriangle (f(x)\, g(x)) + b \blacktriangle (f(x) \,.\, g(x))$
(C) $\blacktriangle [af(x) + bg(x)] = a \Delta f(x) + b \Delta g(x)$
(D) $\blacktriangle [af(x) + bg(x)] = a\Delta(f(x)\, g(x)) + b \Delta (f(x)\, g(x))$

79. Let $f(x)$ be a function which takes the values $y_0, y_1, \ldots\ldots., y_n$ for $x = x_0, x_1, \ldots\ldots., x_n$ respectively and $[a, b]$ be an interval divided into n subintervals of width h such that :
$x_0 = a, x_1 = x_0 + n, \ldots\ldots, x_n = x_0 + nh = b.$
Then the error in the trapezoidal rule for integration is of order :
(A) h
(B) h^2
(C) h^3
(D) cannot determine

80. The value of $\Delta^2 f(x)$, where $f(x) = ab^{cx}$ is :
(A) $(b^{ch} - 1)^2\, ab^{cx}$
(B) $(b^{ch} + 1)\, ab^{cx}$
(C) $(ab^{cx})^2$
(D) $(b^{ch} + 1)^2\, ab^{cx}$

81. Let $y = f(x)$ take the values $y_0, y_1, ..., y_n$ for $a = x_0$, $x_1 = x_0 + h$, ..., $x_n = x_0 + nh = b$. Then the value of :

$$\int_a^b f(x)\, dx$$

by Simpson's $\frac{1}{3}$-rule is :

(A) $\frac{h}{3}[y_0 + 4(y_1 + y_3 + ... + y_{n-1}) + 2(y_2 + y_4 + ... + y_{n-2}) + y_n]$

(B) $\frac{h}{8}[y_0 + y_n + 4(y_1 + y_3 + ... + y_{n-1}) + 2(y_2 + y_4 + ... + y_{n-2})]$

(C) $\frac{h}{3}[4(y_0 + y_2 + ... + y_n) + 2(y_1 + y_3 + ... + y_{n-1})]$

(D) $\frac{h}{8}[4(y_0 + y_2 + ... + y_n) + 2(y_1 + y_3 + ... + y_{n-1})]$

82. The extremal of the functional

$$I(r(\theta)) = \int_{\theta_0}^{\theta} \sqrt{r^2 + r'^2}\, d\theta, \quad r' = \frac{dr}{d\theta}$$

is :

(A) a catenary

(B) a cycloid

(C) a straight line

(D) an arc of the great circle

83. The shortest distance between two points in a 3-dimensional Euclidean space is a curve of intersection of :

(A) two surfaces

(B) two planes

(C) a surface by a plane

(D) a sphere of radius r by the plane through the centre of the sphere

84. The time taken by a particle moving along a curve $y = y(x)$ with velocity $v = x$ from the point (0, 0) to (1, 1) is minimum if the curve is a :

(A) Circle with its centre on the x-axis

(B) Cycloid

(C) Catenary

(D) Circle with its centre on the y-axis

85. The extremals of the functional

$$\mathrm{I}(y(x)) = \int_{x_1}^{x_2} f(x, y')\, dx$$

is obtained as the solution of the equation :

(A) $f_{y'x}\, y' + f_{y'} = 0$

(B) $f_{y'y'}\, y'' + f_{y'x} = 0$

(C) $y' f_{y'} - f = \text{constant}$

(D) $f_{y'x} = 0$

86. The functional

$$\mathrm{I}(y(x)) = \int_{0}^{\pi/4} (y^2 - y'^2)\, dx$$

satisfying $y(0) = 0$ attains its extremal on the curve :

(A) $y = b \sin x$

(B) $y = e^x$

(C) $y = \cos x$

(D) $y = 0$

87. Which of the following functions is coercive ?

(A) $f : \mathbf{R}^2 \to \mathbf{R}$, $f(x, y) = 3x - 4y$

(B) $f : \mathbf{R}^2 \to \mathbf{R}$, $f(x, y) = x$

(C) $f : (0, \infty) \to \mathbf{R}$ $f(x) = y_x$

(D) $f : \mathbf{R}^2 \to \mathbf{R}$, $f(x, y) = x^2 + y^2$

88. For the homogeneous integral equation

$$x(t) = \lambda \int_0^1 \left(s\sqrt{t} - t\sqrt{s}\right) x(s)\, ds,$$

consider the following statements.

(I) This equation does not have real eigenvalues.

(II) The eigenvalues of this equation are 1 and –1.

Then :

(A) Only (I) is true

(B) Only (II) is true

(C) Both (I) and (II) are true

(D) Both (I) and (II) are false

89. The solution of the integral equation

$$\frac{\pi t}{2} = \int_0^t \frac{1}{\sqrt{t-s}}\, x(s)\, ds$$

is :

(A) $x(t) = \dfrac{1}{\sqrt{t}}$

(B) $x(t) = \sqrt{t}$

(C) $x(t) = 1$

(D) $x(t) = t$

90. The boundary value problem

$$y'' + \lambda y = x, \quad y(0) = 0, \quad y'(1) = 0$$

is equivalent to the Fredholm integral equation.

(A) $y(x) = \frac{1}{6}(x^3 - 3x) + \lambda \int_0^1 k(x, t)\, y(t)\, dt$

(B) $y(x) = \frac{1}{6}(x^2 - 2x) + \lambda \int_0^1 x\, k(x, t)\, y(t)\, dt$

(C) $y(x) = \frac{1}{3}(x - 3x^2) + \lambda \int_0^1 (x - t)\, k(x, t)\, y(t)\, dt$

(D) $y(x) = (x - 2x^2) + \lambda \int_0^1 e^{(x-t)}\, k(x, t)\, y(t)\, dt$

where $k(x, t) = \begin{cases} t; & 0 < t < x \\ x; & x < t < 1 \end{cases}$

91. Eigenvalues of the Fredholm integral equation,

$$x(t) = 1 + \lambda \int_0^{\pi} \cos(t + s)\, x(s)\, ds$$

are :

(A) $2/\pi, \ -2/\pi$

(B) $\pi, \ -\pi$

(C) $1, \ -1$

(D) $\sqrt{2}, \ -\sqrt{2}$

92. The resolvent kernel for Volterra integral equation with kernel :

$$k(t, s) = a^{t-s}, \quad a > 0$$

is :

(A) $R(t, s; \lambda) = a^{\lambda(t-s)} e^{(t-s)}$

(B) $R(t, s; \lambda) = a^{t+s} e^{\lambda(t+s)}$

(C) $R(t, s; \lambda) = a^{t-s} . e^{\lambda(t-s)}$

(D) $R(t, s; \lambda) = a^{t+s} e^{\lambda(t-s)}$

93. For the integral equation

$$x(t) = \frac{1}{2}\int_t^1 (t\sqrt{s} - s\sqrt{t})\, x(s)\, ds$$

consider the following statements. The above integral equation is :

(I) Fredholm integral equation of second kind.

(II) Volterra integral equation of first kind.

Then :

(A) Only (I) is true

(B) Only (II) is true

(C) Both (I) and (II) are true

(D) Both (I) and (II) are false

94. A particle is thrown horizontally from the top of a building of height h with initial velocity u. Neglecting all other forces except the gravity and the air resistance which is proportional to the velocity of the particle, then which one of the following is true ?

(A) The Hamiltonian H represents the total energy

(B) Angular momentum is conserved

(C) Hamiltonian H is conserved

(D) The total energy E is conserved

95. A particle of mass m is constrained to move on the arc of a parabola $x^2 = 4ay$, where y is vertical axis, under gravity. Then the canonical momentum is given by :

(A) $m\dot{x}$

(B) $m(\dot{x} - 2a\dot{y})$

(C) $mx\left(\frac{\dot{x}^2}{4a^2}\right)$

(D) $m\dot{x}\left(1 + \frac{x^2}{4a^2}\right)$

96. A particle of mass m is constrained to move on a horizontal xy-plane which is rotating about the vertical z-axis with angular velcoity ω. If T is the total kinetic energy of the particle, then $\dot{x}\frac{\partial T}{\partial \dot{x}} + \dot{y}\frac{\partial T}{\partial \dot{y}}$ is equal to :

(A) 2T

(B) $2T_2 + T_1$

(C) $2T_2 + T_1 + T_0$

(D) $T_2 - 2T_1 - T_0$

where $T = T_2 + T_1 + T_0$,

$$T_2 = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2), \quad T_1 = m\omega(x\dot{y} - y\dot{x}),$$

$$T_0 = \frac{1}{2}m\omega^2(x^2 + y^2)$$

97. If $L(q_j, \dot{q}_j, t)$ and $H(p_j, q_j, t)$ are respectively the Lagrangian and the Hamiltonian of a dynamical system, then the Lagrange's equations of motion $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right) - \frac{\partial L}{\partial q_j} = 0$ can also be represented as :

(A) $\frac{\partial L}{\partial t} + \frac{dH}{dt} = 0$

(B) $\frac{\partial L}{\partial q_j} + \frac{d}{dt}\left(L - \sum_j \dot{q}_j \frac{\partial L}{\partial \dot{q}_j}\right) = 0$

(C) $\frac{\partial L}{\partial t} + \frac{d}{dt}\left(L - \sum_j \dot{q}_j \frac{\partial L}{\partial \dot{q}_j}\right) = 0$

(D) $\frac{\partial L}{\partial \dot{q}_j} - \frac{d}{dt}\left(L - \sum_j p_j \dot{q}_j\right) = 0$

98. A particle of mass m is thrown upward from the surface of the earth with initial velocity u making an angle θ with the x-axis. The instant of time at which the acceleration and the velocity of the particle are perpendicular to each other is :

(A) $\frac{u \cos\theta}{g}$

(B) $\frac{2u \sin\theta}{g}$

(C) $\frac{2u \cos\theta}{g}$

(D) $\frac{u \sin\theta}{g}$

99. If p_j and $p_j^{'}$ are the components of the generalised momenta corresponding to the Lagrangians L and L' respectively, where

$$L' = L + \frac{d}{dt} F(q_j, t),$$

then :

(A) $p_j^{'} = p_j$

(B) $p_j^{'} = p_j + \frac{\partial F}{\partial q_j}$

(C) $p_j^{'} = p_j + \frac{\partial F}{\partial t}$

(D) $p_j^{'} = p_j + \frac{d}{dt}\left(\frac{\partial F}{\partial q_j}\right)$

100. Let L_x, L_y, L_z be the components of angular momentum of a rigid body with one point fixed. If L_x is of the form

$L_x = I_{xx} W_x + I_{xy} W_y + I_{xz} W_z$,

where I_{xx} is the moment of inertia about x-axis, then :

(A) $I_{xx} = \sum_i m_i \left(x_i^2 + y_i^2\right)$

(B) $I_{xx} = \sum_i m_i \left(x_i^2 + z_i^2\right)$

(C) $I_{xx} = \sum_i m_i \left(y_i^2 + z_i^2\right)$

(D) $I_{xx} = -\sum_i m_i x_i^2$

SECTION III

101. The arithmetic mean and product of three geometrically progressed numbers are 7 and 216. The arithmetic mean of first and last number is :
   (A) 7.5
   (B) 6.5
   (C) 5.5
   (D) 8.5

102. If in a questionnaire, place of residence is asked with options as suburbs, city, town, metro, villages, then appropriate scale of measurement is :
   (A) Ratio scale
   (B) Ordinal scale
   (C) Interval scale
   (D) Nominal scale

103. The upper quartile of the data is 7.75 and coefficient of quartile deviation 0.55. The interquartile range is :
   (A) 5.00
   (B) 5.25
   (C) 5.50
   (D) 5.75

104. The population of Nicosia is 75% Greek and 25% Turkish. 20% of the Greeks and 10% of the Turks speak English. A visitor to the town meets someone who speaks English. What is the probability that he is a Greek ?
   (A) $\frac{3}{7}$
   (B) $\frac{4}{7}$
   (C) $\frac{5}{7}$
   (D) $\frac{6}{7}$

105. Suppose that X and Y are i.i.d. random variables with c.d.f. F which is binomial with parameters n and p. Then, $P(x = j \mid x + y = k)$ is equal to :
   (A) j/k, for $j = 0, 1, 2, ..., k$
   (B) $\binom{k}{j} p^j q^{k-j}$, for $j = 0, 1, 2, ..., k$
   (C) $\binom{k}{j} (1/2)^k$, for $j = 0, 1, 2, ..., k$
   (D) $\binom{n}{j}\binom{n}{k-j} / \binom{2n}{n}$, for $j = 0, 1, 2, ..., k$

106. Let $F(x) = P[X \le x]$ be the distribution function of a random variable X.

Define $D_n = \left\{u \mid F(u) - F(u-) \ge \frac{1}{n}\right\}$, $n = 1, 2, 3, ...$

Then :

(A) D_n cannot contain more than n elements

(B) D_n is not empty set for all $n \ge 1$

(C) D_2 always contains 2 elements

(D) D_1 is always empty set

107. Which of the following statements is not true ?

(A) If A_1 and A_2 are two independent events, then A_1^c and A_2^c are also independent events

(B) If $\{A_n, n \ge 1\}$ is a sequence of independent events such that $\sum_{n=1}^{\infty} P(A_n) = \infty$, then $P[A_n$ occurs infinitely often$] = 1$

(C) An event A that is independent of itself if P(A) = 1

(D) An event whose probability is either zero or one is independent of every event

108. A man is known to speak truth 3 out of 4 times. He throws a fair die and reports it is a six. Then, the probability that it is actually six is equal to :

(A) 1/8

(B) 5/8

(C) 2/7

(D) 3/8

109. Three software engineers A, B, C are being considered for selection as project leader. The probabilities of their selection are in the ratio 1 : 3 : 6. If A is selected, the probability that she will use R software is 0.8. The corresponding probabilities for B and C are respectively 0.5 and 0.3. The following are two statements :

(I) The probability that R software will be used for the new project is 0.41.

(II) If R software is used, the probability that either A or B is the project leader is 23/41.

Which of the following is *true* ?

(A) Both (I) and (II) are true

(B) Only (I) is true

(C) Only (II) is true

(D) Neither (I) nor (II) is true

110. Suppose that G_1, G_2 and G_3 are functions defined by :

$$G_1(x) = \frac{pF(x)}{1-qF(x)}; \quad G_2(x) = \frac{F(x)}{p+qF(x)};$$

$$G_3(x) = p + qF(2),$$

where $0 < p < 1$, $0 < q < 1$ and $p + q = 1$ and F is a probability distribution function then, which of the following is true ?

(A) G_1, G_2 and G_3 are distribution functions

(B) Only G_1 and G_2 are distribution functions

(C) Only G_1 and G_3 are distribution functions

(D) None of the G_1, G_2, G_3 are distribution functions

111. Suppose that X is a random variable with c.d.f. F. Then, the distribution F is not symmetric around its median when :

(A) F is binomial with parameters n and ½

(B) F is uniform over (a, b)

(C) F is exponential with location parameter θ

(D) F is Cauchy with location parameter θ

112. Suppose X is a non-negative random variable whose mean exists. The following are three statements :

(I) $E(2\log X) \leq 2E(\log X)$

(II) $E(\exp(-2X)) \geq \exp(-2E(X))$

(III) $E\left(X^{\frac{1}{5}}\right) \geq (E(X))^{\frac{1}{5}}$

Which of the following options is true ?

(A) Only (II) is true

(B) Only (II) and (III) are true

(C) Only (III) is true

(D) None of (I), (II) and (III) is true

113. Which of the following is *not* a characteristic function ?

(A) $\phi_1(t) = \phi\left(\frac{t}{a}\right)\exp(ibt)$, $a \neq 0$, $b \in R$ and ϕ is a characteristic function

(B) $\phi_1(t) = \phi(-t)$, where ϕ is a characteristic function

(C) $\phi(t) = \frac{1}{1+t^4}$, $t \in R$

(D) $\phi(t) = \sum_{j=0}^{\infty} a_j \phi_j(t)$, $a_j \geq 0$ and $\sum_{j=0}^{\infty} a_j = 1$, and ϕ_j is a characteristic function, $j = 0, 1, 2, \ldots\ldots$

114. Suppose $\{A_n, n \geq 1\}$ is a sequence of events on a probability space $(\Omega, \mathcal{A}, P)$. Which of the following is always true ?

(A) $\sum_{n\geq1} P(A_n) < \infty \Rightarrow P(\liminf A_n) = 0$

(B) $\sum_{n\geq1} P(A_n) = \infty \Rightarrow P(\liminf A_n) = 1$

(C) $P(\limsup A_n) = 0 \Rightarrow \sum_{n\geq1} P(A_n) < \infty$

(D) $P(\limsup A_n) = 1 \Rightarrow \sum_{n\geq1} P(A_n) = \infty$

115. Suppose $X_n \to X$ almost surely. Suppose Y_n and Y are defined as $Y_n = X_n^3 + 2X_n^2 + 3X_n + 2$ and $Y = X^3 + 2X^2 + 3X + 2$. The following are three statements :

(I) $Y_n \to Y$ almost surely

(II) $Y_n \to Y$ in probability

(III) $Y_n \to Y$ in law.

Which of the following is true ?

(A) Only (I) is true

(B) Only (I) and (II) are true

(C) Only (II) and (III) are true

(D) All three are true

116. Suppose X is a real random variable on a probability space $(\Omega, \mathcal{A}, P)$ and $\{A_n, n \geq 1\}$ is a sequence of sets in $\mathcal{A}$ such that :

$$A_n \to \phi \text{ as } n \to \infty$$

(A) $XI_{A_n} \to 0$ in probability

(B) $XI_{A_n} \to X$ in probability

(C) $XI_{A_n} \to 1$ in probability

(D) $XI_{A_n} \to X$ in law

117. Suppose as $n \to \infty$, $X_n \to 3$ in probability and $Y_n \to Y$ in distribution. Which of the following is true as $n \to \infty$?

(A) $(X_n + Y_n) \to 3 + Y$ in probability

(B) $\dfrac{n(X_n - 4) + Y_n}{n} \to -1$ in probability

(C) $X_n Y_n + Y_n \to 4Y$ in probability

(D) $\dfrac{n(Y_n - 4) + X_n}{n} \to -4$ in probability

118. Let $\{X_n, n \geq 1\}$ be a sequence of random variables such that as $n \to \infty$, $X_n \to X$ in distribution. Which of the following is not always true as $n \to \infty$?

(A) $E[\cos(X_n) - \cos(X)] \to 0$

(B) $E[X_n^2 - X^2] \to 0$

(C) $E\left[\dfrac{X_n^2}{1+X_n^2} - \dfrac{X^2}{1+X^2}\right] \to 0$

(D) $E\left[e^{i(X_n+2)} - e^{i(X+2)}\right] \to 0$, where $i = \sqrt{-1}$

119. The following are the transition probability matrices of the Markov chains with state space $S = \{1, 2\}$:

$$P_1 = \begin{array}{cc} & \begin{array}{cc} 1 & 2 \end{array} \\ \begin{array}{c} 1 \\ 2 \end{array} & \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \end{array} \qquad P_2 = \begin{array}{cc} & \begin{array}{cc} 1 & 2 \end{array} \\ \begin{array}{c} 1 \\ 2 \end{array} & \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \end{array}$$

$$P_3 = \begin{array}{cc} & \begin{array}{cc} 1 & 2 \end{array} \\ \begin{array}{c} 1 \\ 2 \end{array} & \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{2}{3} & \frac{1}{3} \end{bmatrix} \end{array} \qquad P_4 = \begin{array}{cc} & \begin{array}{cc} 1 & 2 \end{array} \\ \begin{array}{c} 1 \\ 2 \end{array} & \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ 0 & 1 \end{bmatrix} \end{array}$$

Which Markov chains have a stationary distribution given by $\left(\dfrac{1}{2}, \dfrac{1}{2}\right)$?

Markov chains with transition probability matrices.

(A) P_1, P_3

(B) P_1, P_2

(C) P_1, P_4

(D) P_2, P_3

120. Suppose $\{X_n, n \geq 0\}$ is a Markov chain with state space $S = \{1, 2, 3, 4\}$ and the transition probability matrix P given by :

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} \;1\; & \;\;2\;\; & \;\;3\;\; & \;\;4\;\; \end{array} \\ \begin{bmatrix} \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & \frac{1}{3} & 0 & \frac{2}{3} \\ \frac{2}{3} & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{4} & 0 & \frac{3}{4} \end{bmatrix} \end{array}$$

Suppose f_{ij} denotes the probability of first visit to j from i. The following are four statements :

(I) $f_{12} = 0$, (II) $f_{24} = 1$, (III) $f_{13} = 1$, (IV) $f_{33} < 1$.

Which of the following is true ?

(A) Only (I) and (II) are true

(B) Only (I) and (III) are true

(C) Only (I) and (IV) are true

(D) Only (I), (II) and (III) are true

121. Suppose in a Markov chain, f_{22} denotes the probability of the first return to state 2 and μ_2 denotes the mean recurrence time to state 2. Suppose $\sum_{n \geq 1} p_{22}^{(n)} = 2.45$. Then which of the following options is true ?

(A) $f_{22} < 1$

(B) $f_{22} = 1$

(C) $\mu_2 < \infty$

(D) $\lim_{n \to \infty} p_{22}^{(n)} > 0$

122. Suppose $f_{ij}^{(n)}$ denotes the probability of the first visit to j from i in n steps. Suppose the Markov chain is periodic with period 2. The following are three statements :

(I) $f_{ii}^{(2)} = p_{ii}^{(2)} - p_{ii}^{(1)} f_{ii}^{(1)}$

(II) $f_{ii}^{(4)} = p_{ii}^{(4)} - p_{ii}^{(2)} f_{ii}^{(2)}$

(III) $f_{ii}^{(7)} = 0$

Which of the following is true ?

(A) Only (I) and (II) are true

(B) Only (I) and (III) are true

(C) Only (III) is true

(D) All three are true

123. Which of the following options is correct ?

Messages arrive on a mobile phone according to the Poisson process with rate of 5 messages per hour. The probability that the first message in the afternoon arrives by 1 : 20 pm is :

(A) $\frac{20}{3} e^{-\frac{20}{3}}$

(B) $e^{-\frac{20}{3}}$

(C) $100\, e^{-100}$

(D) $\frac{5}{3} e^{-\frac{5}{3}}$

124. Which of the following is true ?

Suppose $\{X(t), t \geq 0\}$ is a Yule-Furry process with birth rate λ and $X(0) = 1$. Then the distribution of $X(t)$ is geometric with success probability p, where :

(A) $p = e^{-\lambda t}$ and support {0, 1, 2, ...,}

(B) $p = e^{-\lambda t}$ and support {1, 2, 3, ...,}

(C) $p = 1 - e^{-\lambda t}$ and support {1, 2, ...,},}

(D) $p = 1 - e^{-\lambda t}$ and support {0, 1, 2, ...,},}

125. If $p(x) = kp^x$, where $0 < p < 1$ and $x = 0, 2, 4, 6, \ldots..$, then for what value of $kp(x)$ becomes probability mass function of a r.v. ?

(A) p

(B) $1 - p$

(C) $1 - p^2$

(D) p^2

126. If $P_X(t)$ is the probability generating function (*pgf*) of random variable X, then *pgf* of $Y = 3X + 2$ is :

(A) $t^2 P_X(t^3)$

(B) $P_X(t^3)$

(C) $t^3 P_X(t^2)$

(D) $2t P_X(t)$

127. Suppose that X is a random variable with c.d.f. F, where $F(x) = 0.7\ \Delta_1(x) + 0.3\ N_{(0,\ 1)}(x)$, where Δ_1 is the degenerate distribution function degenerate at 1 and $N_{(0,\ 1)}$ is the standard normal distribution function. Then, which of the following statements is true ?

(A) E(X) = 0.7 and variance (X) = 1

(B) E(X) = 0.7 and variance (X) = 0.30

(C) E(X) = 0.7 and variance (X) = 0.51

(D) E(X) = 0 and variance (X) = 0.30

128. Suppose that X is a random variable with cumulative distribution function F, probability density function f, and moment generating function M. If M satisfies the condition :

$$e^{-5t}\,M(t) = e^{5t}\,M(-t),\quad \forall\ t \geq 0,$$

then which of the following statements is FALSE ?

(A) $f(x+5) = f(5-x)\ \forall\ x$

(B) $f(x-5) = f(5-x)\ \forall\ x$

(C) 5 is the median of F

(D) X – 5 and 5 – X are identically distributed

129. Suppose that X and Y are two random variables with c.d.f. F_1 and F_2 respectively, where

$$F_1(x) = \begin{cases} 0 & \text{if} \quad x < 0 \\ 2/3 & \text{if} \quad 0 \leq x < 1 \\ 1 & \text{if} \quad x \geq 1 \end{cases} \quad \text{and}$$

$$F_2(x) = \begin{cases} 0 & \text{if} \quad x < 0 \\ \sqrt{x} & \text{if} \quad 0 \leq x < 1 \\ 1 & \text{if} \quad x \geq 1 \end{cases}$$

Then, which of the following statements is true ?

(A) E(X) = E(Y)

(B) E(X) = 2/3 and E(Y) = 1/3

(C) E(X) = 1/3 and E(Y) = 1/2

(D) E(X) = 1/3 and E(Y) = 1/6

130. Suppose that X_K, K = 1, 2, 3, 4 are independent and identically distributed random variables such that X_1 has exponential distribution with scale parameter 1. Then, which of the following statements is/are true ?

(I) $\frac{X_1 + X_2}{X_3 + X_4}$ has a F-distribution

(II) $\frac{X_1 + X_2}{X_1 + X_2 + X_3}$ has a β-distribution

(A) Both (I) and (II) are true

(B) Only (I) is true

(C) Only (II) is true

(D) Neither (I) nor (II) is true

131. Let (X_1, X_2, X_3) be a random sample from $B(1, p)$. Which one of the following statistics is not sufficient for p ?

(A) $X_1 + X_2 + X_3$

(B) $(X_1, X_2 + X_3)$

(C) $(X_1 + X_2, X_3)$

(D) $X_1 - X_2 + X_3$

132. For testing H_0 against H_1, if test function $\phi(\underline{X})$ is UMP, then :

(A) $\phi(\underline{X})$ is not unbiased

(B) $\phi(\underline{X})$ is unbiased

(C) $\phi(\underline{X})$ has power less than its size

(D) $\phi(\underline{X}) \equiv 1$

133. Consider the following hypothesis testing problems P_1 and P_2 for μ based on sample from $N(\mu, 1)$.

$P_1 : H_0 : \mu = \mu_0$ *Vs* $H_1 : \mu > \mu_0$

$P_2 : H_0 : \mu = \mu_0$ *Vs* $H_1 : \mu \neq \mu_0$

The UMP level α test exists :

(A) For both P_1 and P_2

(B) For P_1 but not for P_2

(C) For P_2 but not for P_1

(D) Neither for P_1 nor for P_2

134. Suppose that $X_1, X_2, \ldots\ldots, X_n$ are i.i.d. random variables such that X_1 has normal distribution with mean θ and variance $\theta > 0$. Then, which of the following is a minimal sufficient statistic for θ ?

(A) $\overline{X}$

(B) $\sum_{i=1}^{n}(X_i - \overline{X})^2$

(C) $\left(\overline{X}, \sum_{i=1}^{n}(X_i - \overline{X})^2\right)$

(D) $\sum_{i=1}^{n} X_i^2$

135. Suppose that $x_1 = -2$, $x_2 = 1$, $x_3 = 3$, $x_4 = -4$ be the observed values of a random sample from the c.d.f. with density,

$$f(x, \theta) = \frac{e^{-x}}{e^{\theta} - e^{-\theta}}, \quad -\theta < x < \theta, \quad \theta > 0.$$

Then, the maximum likelihood estimator of θ is :

(A) 3

(B) 0.5

(C) 4

(D) 1.5

136. If T_1 is an unbiased estimator of θ and T is a minimal sufficient statistic for θ, then which of the following statements is NOT true ? Suppose $T^* = E[T_1 | T]$.

(A) Var $(T^*) \leq$ Var (U) for every unbiased estimator U of θ and T a complete sufficient statistic

(B) T^* represents the procedure of Rao-Blackwellisation

(C) Var $(T^*) \leq$ Var (U), for every unbiased estimator U of θ

(D) Var $(T^*) \leq$ Var (T_1)

137. Let $X_1, X_2, \ldots\ldots, X_{2n+1}$ be a random sample from a uniform distribution on the interval $(\theta - 1, \theta + 1)$.

Let

$T_1 = \overline{X}$ (sample mean on $2n + 1$ observations)

$T_2 = \tilde{X}$ (sample median on $2n + 1$ observations)

$T_3 = \dfrac{T_1 + T_2}{2}$

be three estimators of θ. Then which of the following statements is correct ?

(A) T_1 is consistent but T_2 is not consistent for θ

(B) T_1 is consistent for θ

(C) Only T_1 is unbiased for θ

(D) T_2 is sufficient statistic for θ

138. Which of the following distributions belong to the one parameter exponential family ?

(A) Uniform $U(0, \theta)$ distribution

(B) Gamma distribution with scale parameter 3 and shape parameter $\lambda \in \{2, 3, 4, 5, \ldots\ldots,\}$

(C) $N(\theta, \theta^2)$ $\theta > 0$

(D) Laplace distribution with location parameter 0 and scale parameter $\theta > 0$

139. Suppose $\{X_1, X_2, \ldots\ldots, X_n\}$ are independent and identically distributed random variables each following $N(\mu, \mu^2)$ distribution, $\mu \in R$. Which of the following statements is NOT true ?

(A) $S_n^2 = \frac{1}{n}\sum\left(X_i - \bar{X}\right)^2$ is a consistent estimator of μ^2

(B) $S_n = \sqrt{S_n^2}$ cannot be a consistent estimator of μ

(C) $\bar{X}_n$ is a consistent estimator of μ

(D) Sample median is a consistent estimator of μ

140. Suppose $\{(X_1, Y_1), (X_2, Y_2), \ldots\ldots, (X_n, Y_n)\}$ is a random sample from bivariate normal distribution with correlation coefficient ρ. It is known that :

$$\sqrt{n}\,(r_n - \rho) \xrightarrow{d} z \sim N\left(0, (1-\rho^2)^2\right)$$

distribution, where r_n denotes the sample correlation coefficient. Which of the following transformation $g(\cdot)$ results in :

$$\sqrt{n}\left(g(r_n) - g(\rho)\right) \xrightarrow{d} z \sim N(0, 1) \ ?$$

(A) $g(\rho) = \frac{1}{2}\log\frac{1+\rho}{1-\rho}$

(B) $g(\rho) = \frac{1}{2}\log\frac{1-\rho}{1+\rho}$

(C) $g(\rho) = \log\frac{1+\rho}{1-\rho}$

(D) $g(\rho) = \log\frac{1+2\rho}{1-2\rho}$

141. Suppose $\{X_1, X_2, ..., X_n\}$ is a random sample from an exponential distribution with mean θ. Then $100(1-\alpha)\%$ asymptotic confidence interval for θ based on sample mean $\bar{X}_n$:

(A) is $\left(\frac{\sqrt{n}\,\bar{X}_n}{z_{\alpha/2}+\sqrt{n}}, \frac{\sqrt{n}\,\bar{X}_n}{z_{1-\alpha/2}+\sqrt{n}}\right)$

(B) is $\left(\frac{\sqrt{n}\,\bar{X}_n}{z_{1-\alpha/2}+\sqrt{n}}, \frac{\sqrt{n}\,\bar{X}_n}{z_{\alpha/2}+\sqrt{n}}\right)$

(C) is $\left(\frac{\bar{X}_n}{z_{1-\alpha/2}+\sqrt{n}}, \frac{\bar{X}_n}{z_{\alpha/2}+\sqrt{n}}\right)$

(D) Cannot be determined

142. A test in which probability of rejecting H_0 when it is not true is more than that of rejecting it when it is true, is said to be :

(A) Unbiased test

(B) Biased test

(C) Consistent test

(D) Uniformly most powerful test

143. In a multinomial distribution with k cells and probability vector $\underline{p}$, we want to test $H_0 : \underline{p} = \underline{p}_0$ against $H_1 : \underline{p} \neq \underline{p}_0$ where $\underline{p}_0$ is a specified vector. Suppose o_i and e_i denote the observed and expected frequencies when the experiment is repeated n times. Which of the following is true ?

The asymptotic null distribution of :

(A) The likelihood ratio test statistic is χ^2_{n-k}

(B) Karl Pearson's test statistic $\sum_{i=1}^{k}\frac{(o_i-e_i)^2}{e_i}$ is χ^2_{k-1}

(C) Karl Pearson's test statistic $\sum_{i=1}^{k}\frac{(o_i-e_i)^2}{e_i}$ is χ^2_k

(D) $\sum_{i=1}^{k}\frac{(o_i-e_i)^2}{o_i}$ is χ^2_k

144. Which one of the following non-parametric test is analogous to the chi-square test of goodness of fit ?

(A) Mann-Whitney test

(B) Kolmogorov-Smirnov test

(C) Median test

(D) Wilcoxon test

145. Which of the following statements about the Bayes factor is true ?

(I) Bayes factor involves marginal likelihood.

(II) Bayes factor expresses the support given by the data for one or other of the models.

(III) Bayes factor involves the conditional likelihood

(A) Only (I) and (III)

(B) Only (I) and (II)

(C) Only (II) and (III)

(D) All are true

146. Given λ, X_1, X_2, X_3, X_4 are independent Poisson random variables with mean λ. Suppose the prior distribution of λ is exponential with mean one. Then the posterior density of λ given $X_i = K_i$, $i = 1, 2, 3, 4$ is :

(A) Gamma $\left(\sum_{i=1}^{4} k_i + 1, 8\right)$

(B) Gamma $\left(\sum_{i=1}^{4} k_i, 8\right)$

(C) Gamma $\left(\sum_{i=1}^{4} k_i + 1, 4\right)$

(D) Exponential with mean $1/\sum_{i=1}^{4} k_i$

147. Consider a linear model $Y_{n\times1} = X_{n\times p}\theta_{p\times1} + \epsilon_{n\times1}$ with $E(\epsilon) = O_{n\times1}$ and $Cov(\epsilon) = \sigma^2 I_n$. If C is a $p \times 1$ vector such that $C'\theta$ is not estimable, then which of the following statement is always true ?

(A) $C'\theta$ has a consistent but biased estimator

(B) $C'\theta$ cannot be estimated with smallest possible variance

(C) $C'\theta$ has a linear biased estimator

(D) $C'\theta$ has a non-linear unbiased estimator

148. In a linear model $Y_{4\times1} = X_{4\times3}\,\theta_{3\times1} + \in_{4\times1}$ $E(Y_1) = E(Y_3) = \theta_1 + \theta_2$ and $E(Y_2) = E(Y_4) = \theta_1 - \theta_3$. Hence which of the following statement is true ?

(A) $\theta_1, \theta_2, \theta_3$ are all estimable

(B) $\theta_1 + \theta_2 + \theta_3$ is estimable

(C) $\theta_1 + \theta_3$ is estimable

(D) $\theta_2 + \theta_3$ is estimable

149. In the following two-way ANOVA table the F ratio corresponding to the block source of variable is 2.5.

Source	***df***	***ss***
Treat	3	2.5
Block	2	*a*
Error	–	9.0
Total	11	*b*

Hence *a* the block sum of squares and *b* total sum of squares are :

(A) $a = 5.0$ and $b = 7.5$

(B) $a = 7.5$ and $b = 19$

(C) $a = 5.0$ and $b = 16.5$

(D) $a = 7.5$ and $b = 10.00$

150. In case of ANOVA with standard assumption, the test of hypothesis tests :

(A) equality of means

(B) equality of means when variances are all equal

(C) equality of variances when means are all equal

(D) equality of means and variances

151. In a simple linear regression, the prediction of the response variable y.

(A) does not depend on the regression X

(B) will have the same variation irrespective of the values of X

(C) will have larger variation as values of X increase

(D) will have larger variation as the distance between the regressor X and its mean $\bar{X}$ increases

152. In case of a multiple linear regression setup $\underline{Y} = X\underline{B} + \underline{\varepsilon}$ with $\text{cov}(\underline{\varepsilon}) = \sigma^2 I_n$, $\sigma^2 > 0$. Let MReg ss MError ss and MTotal ss stand for mean regression sum of squares, mean error sum of squares and mean total sum of squares respectively. Which of the following statements is true ?

(A) MReg ss and MError ss are unbiased estimators of σ^2, MTotal ss is a biased estimator of σ^2

(B) all three mean ss are unbiased estimators of σ^2

(C) all three mean ss are biased estimator of σ^2

(D) MError ss is an unbiased estimator of σ^2, MReg ss and MTotal ss are biased estimator of σ^2

153. In a multiple linear regression setup, the adjusted coefficient of determination accounts for :

(A) the outliers in the data

(B) the unusually large fitted valves

(C) the number of regressors in the model

(D) none of the above

154. Consider a logistic regression model

$$E(Y) = \frac{\exp(B_0 + B_1X_1 + B_2X_2 + B_3X_3 + B_4X_4)}{1 + \exp(B_0 + B_1X_1 + B_2X_2 + B_3X_3 + B_4X_4)}$$

For the deviance based test to test whether the model fits the data of 40 observations, the degrees of freedom are :

(A) 35

(B) 34

(C) 5

(D) 4

155. $\underset{\sim}{X}_{2\times 1} \sim MN_2\left(\begin{bmatrix}\mu_1\\ \mu_2\end{bmatrix}, \begin{bmatrix}1 & \rho\\ \rho & 1\end{bmatrix}\right)$, then $\operatorname{var}(X_1+X_2 \mid X_1-X_2=Y)$ is equal to :
(A) $1 + \rho$
(B) $\rho(1 + \rho)$
(C) 2
(D) $2(1 + \rho)$

156. Suppose $\underset{\sim}{X} = \begin{pmatrix}X_1\\ X_2\\ X_3\end{pmatrix}_{3\times 1}$ Multivariate Normal$_3\left(\begin{bmatrix}1\\ 0\\ 3\end{bmatrix}, \begin{bmatrix}2 & 1 & 0.5\\ 1 & 4 & 1\\ 0.5 & 1 & 1\end{bmatrix}\right)$.
Hence the probability distribution of $\begin{pmatrix}X_1\\ X_2\end{pmatrix}$ given $X_3 = 2$.
(A) is Normal$_2$ $\left(\begin{bmatrix}0.5\\ 1\end{bmatrix}, \begin{bmatrix}2 & 0.5\\ 0.5 & 3\end{bmatrix}\right)$
(B) is Normal$_2$ $\left(\begin{bmatrix}1.5\\ 1\end{bmatrix}, \begin{bmatrix}1.75 & 0.5\\ 0.5 & 3\end{bmatrix}\right)$
(C) is Normal$_2$ $\left(\begin{bmatrix}0.5\\ -1\end{bmatrix}, \begin{bmatrix}1.75 & 0.5\\ 0.5 & 3\end{bmatrix}\right)$
(D) cannot be determined

157. Suppose we have n *iid* observation from multivariate Normal$_p$ $(\underset{\sim}{\mu}, \Sigma)$. The likelihood ratio test to test $H_0 : \underset{\sim}{\mu} = \underset{\sim}{\mu}_0$ against $H_1 : \underset{\sim}{\mu} \neq \underset{\sim}{\mu}_0$ uses :
(A) The t statistic which follows t distribution
(B) The statistic which follows χ^2 distribution
(C) Hotelling T^2 statistic which follows F distribution
(D) None of the above

158. The quadratic form
$$4X_1^2 + X_2^2 + X_3^2 - 2X_1X_2 + 2X_1X_3$$
is :
(A) negative definite
(B) negative semidefinite
(C) positive definite
(D) positive semidefinite

159. Suppose $\underset{\sim}{X}_{3\times1} \sim$ Multivariate Normal$_3$ $(\underset{\sim}{\mu}\ \Sigma)$ and

$$Y = X_1^2 - X_2^2 + X_3^2 + 0.5X_1X_3.$$

Then the probability distribution of y is :

(A) not χ^2

(B) χ^2 with 3 degrees of freedom

(C) χ^2 with 2 degrees of freedom

(D) χ^2 with 1 degree of freedom

160. For three random variables X_1, X_2, X_3 if $\rho_{12} = 0.6$ and $\rho_{13} = 0.8$, then ρ_{23} :

(A) can be < 0

(B) can be any value in [0, 1]

(C) has to be in [–0.96, 0.96]

(D) has to be in [0, 0.96]

161. The Mahalanobis distance between the two Bivariate Normal Population

$$N_2\left(\begin{bmatrix}1\\-1\end{bmatrix}, \begin{bmatrix}1 & 1\\1 & 4\end{bmatrix}\right) \text{ and }$$

$$N_2\left(\begin{bmatrix}2\\-2\end{bmatrix}, \begin{bmatrix}1 & 1\\1 & 4\end{bmatrix}\right)$$

is :

(A) 1/3

(B) 7/3

(C) 10/3

(D) 13/3

162. Given n observations on p variables each, it is decided to carry out the principal component analysis such that the proportion of variation explained by k principal components is at most 0.7.

(A) k is 70% of p

(B) k is 70% of n

(C) k is the smallest value for which k principal components explain at least 70% of total variation

(D) k is the largest value for which at least k principal components explain at least 70% of total variation

163. If population size N is very large, then the approximate value of minimum sample size for estimating population mean with desired accuracy under SRSWOR design is :

(A) $\left(\frac{s_y Z_{\alpha/2}}{d}\right)^{½}$

(B) $\sqrt{N}$

(C) $\frac{(s_y Z_{\alpha/2})^2}{d}$

(D) $\left(\frac{s_y Z_{\alpha/2}}{d}\right)^2$

where α and d are

$$P[|\bar{Y} - \bar{y}| < d] \geq 1 - \alpha$$

164. Consider a population of size N = 6 as {1, 2, 3, 4, 5, 6} with their auxiliary variable values as :

$x_1 = 100$, $x_2 = 200$, $x_3 = 300$,

$x_4 = 400$, $x_5 = 500$ and $x_6 = 500$.

We want to draw a random sample of $n = 3$ using PPSWOR design. Let U_r denote the unit selected at the rth draw for $r = 1, 2, 3$.

Then :

$$P[U_3 = 2 | U_1 = 5, U_2 = 3] =$$

(A) $\frac{1}{6}$

(B) $\frac{1}{10}$

(C) $\frac{2}{11}$

(D) $\frac{2}{13}$

165. For a sampling design $(\mathfrak{U}, \mathfrak{S}, P)$, the Horvitz-Thompson estimator for population total is given by :

(A) $\frac{1}{N}\sum_{i=1}^{N}\frac{y_i T_i}{\pi_i}$

(B) $\sum_{i=1}^{N}\frac{y_i T_i}{\pi_i}$

(C) $\frac{N}{n}\sum_{i=1}^{N}\frac{y_i}{\pi_i}$

(D) $\sum_{i=1}^{N}\frac{y_i}{N\pi_i}$

166. Which of the following relation is true under post stratification ?

(A) $V\left(\bar{Y}_{st}\right)_p > V\left(\bar{Y}_{st,\text{post}}\right)$

(B) $V\left(\bar{Y}_{st}\right)_p < V\left(\bar{Y}_{st,\text{post}}\right)$

(C) $V\left(\bar{Y}_{st,\text{post}}\right) \cong V\left(\bar{Y}_{st}\right)_p$

(D) $V\left(\bar{Y}_{st,\text{post}}\right) = \sum_{h=1}^{L} w_h s_h^2 \left(\frac{1}{M_h} - \frac{1}{n_h}\right)$

167. Which of the following is NOT true for stratified random sampling ?

A larger sample would be required from a stratum if :

(A) stratum size is large

(B) stratum variability is large

(C) sampling cost per unit is large in the stratum

(D) sampling cost per unit is small in the stratum

168. Which of the following statements is true ?

(A) BIBD can be more efficient than RBD

(B) BIBD (r, b, s, k, λ) (standard notation is followed) is symmetrical when $b > r + s - k$

(C) BIBD is always more efficient than RBD

(D) A BIBD can be constructed for any choice of positive integers (r, b, s, k, λ)

169. Which of the following is NOT TRUE for the 'C' matrix of an incomplete block design ?

(I) C matrix is symmetric.

(II) The row sum is equal to column sum for any row and column.

(A) Only (I)

(B) Only (II)

(C) Both (I) and (II)

(D) Neither (I) nor (II)

170. In a BIBD ($r = 7$, $b = 21$, $k = 5$, $s = 15$, $\lambda = 10$), the following are two statements :

(I) Each treatment is repeated 15 times.

(II) The given parameters of the BIBD is valid.

Which of the following is true ?

(A) Only (I)

(B) Only (II)

(C) Both (I) and (II)

(D) Neither (I) nor (II)

171. In a Latin square design involving 5 treatments, the sum of squares due to Rows, Columns and Treatments are respectively 80, 4 and 60. Suppose the F-statistic for testing the equality of treatments is 30, what will be the total sum of squares ?

(A) 175

(B) 146

(C) 150

(D) 158

172. Which of the following statements is/are true ?

(I) A two-way classified data will be orthogonal when the all frequencies are proportional.

(II) In a given set of n values $y_1, y_2, \ldots\ldots, y_n$, the maximum number of mutually orthogonal contrasts among them.

(A) Only (I)

(B) Only (II)

(C) Both (I) and (II)

(D) Neither (I) nor (II)

173. Suppose the lifetime of a system has an absolutely continuous distribution with hazard rate $\lambda(t)$, $t \geq 0$. Which of the following statements is true ?

(A) $\lambda(t)$ is the probability that the system fails at time 't'

(B) $\lambda(t)$ is the conditional probability that the system fails at time 't' given that it has survived upto time 't'

(C) $\lambda(t)$ can be any non-negative function for which $\lim_{t\to\infty} \int_0^t \lambda(u)\, du$ diverges to ∞

(D) $\int_0^\infty \lambda(u)\, du$ need not be 1 but is finite

174. Consider a series system with 3 components. Suppose the component lifetimes are independent but with failure rates λ_1, λ_2 and λ_3. Then the failure rate of the system lifetime :

(A) is $\lambda_1 \lambda_2 \lambda_3$

(B) cannot be obtained from the given information

(C) min $(\lambda_1, \lambda_2, \lambda_3)$

(D) $\lambda_1 + \lambda_2 + \lambda_3$

175. Let $X_1, ..., X_n$ be a random sample from a lifetime distribution F with parameter θ. Let f denote the corresponding probability density function and let T_0 be a fixed time. We observe $X_i = x_i$ only if $x_i \leq T_0$, $i = 1, 2, ..., n$. Let n_u denote the number of observed lifetimes and $x_{(1)} < x_{(2)} < ......$ be the ordered observations. Which of the following is the most appropriate 'likelihood' for estimating θ ?

(A) $\prod_{i=1}^{n} f(x_{(i)})$

(B) $\prod_{i=1}^{n_u} f(x_{(i)})$

(C) $\prod_{i=1}^{n_u} f(x_{(i)}) / (f(T_0))^{n-n_u}$

(D) $\prod_{i=1}^{n_u} f(x_{(i)})\, (1-F(T_0))^{n-n_u}$

176. For the LPP

Min : $4x + 6y + 18z$

s.t. $x + 3y \geq 3;$

$y + 2z \geq 5;$

$x, y, z \geq 0,$

which of the following is *not* a constraint in the corresponding dual ?

(A) $u \leq 4$

(B) $v \leq 9$

(C) $3u + v \geq 6$

(D) $u, v \geq 0$

177. Consider the following LPP

Max : $x_1 + 5/2\ x_2$

Subject to

$5x_1 + 3x_2 = 15;$

$-x_1 + x_2 \leq 1;$

$2x_1 + 5x_2 \leq 10;$

$x_1, x_2 \geq 0$

The problem :

(A) has no feasible solution

(B) has infinitely many optimal solutions

(C) has a unique optimal solution

(D) has an unbounded solution

178. The optimal value of the LPP

Max : $-4x + 5y,$

Subject to :

$y \leq x$ and $x \leq 4$

(A) is 8

(B) is 4

(C) is 0

(D) does not exist

179. Arrival rate at a M|M|1 queuing system is 0.11 per minute, while service rate is 0.33 per minute. Then, which of the following statements is true ?

(I) The probability that a customer has to wait is 0.67.

(II) The probability of waiting for more than 10 minutes is 0.03.

(A) Neither is true

(B) Both are true

(C) Only (I) is true

(D) Only (II) is true

180. A service center consists of a server working at an exponential rate of two services per hour. If customers arrive at a Poisson rate of one per hour, then what is the probability that there are three customers in the long run ?

(A) 1/4

(B) 1/2

(C) 1/16

(D) 1/64

ROUGH WORK

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