**MATHEMATICS**

**PAGEMAKER10**

**Permutation and Combination**

Q1. The value of 2n{1, 3, 5 ................... (2, n – 3) (2n – 1)}

(a) 2n!$|$n!

(b) 2n!$|$2n

(c) n!$|$2n!

(d) none

L1Difficulty1

Qtag Mathematics

Qcreator Pagemaker10

Q2. The number of ways in which the letter of the word TRIANGLE can be arranged such that two vowels do not occur together is

(a) 1200

(b) 2400

(c) 14400

(d) none

L1Difficulty1

Qtag Mathematics

Qcreator Pagemaker10

Q3. The exponent of 3 in 100 !

(a) 33

(b) 44

(c) 48

(d) 52

L1Difficulty1

Qtag Mathematics

Qcreator Pagemaker10

Q4. The No. of times the digit 3 will be written when listing the integer from 1 to 1000.

(a) 369

(b) 300

(c) 271

(d) 302

L1Difficulty1

Qtag Mathematics

Qcreator Pagemaker10

Q5. The No. of ways in which an examiner can assin 30 marks to 8 question, awarding not less than 2 marks to any question is

(a) $21\_{c7}$

(b) $30\_{c16}$

(c) $21\_{c16}$

(d) none

L1Difficulty1

Qtag Mathematics

Qcreator Pagemaker10

Q6. A person goes in for an examination in which there are four persons with a maximum of m marks from each paper. The No. of ways in which one can get 2m marks.

(a) 2m + $3\_{c3}$

(b) $\frac{1}{3}$ (m + 1) (2m2 + 4m + 1)

(c) $\frac{1}{3}$ (m + 1) (2m2 + 4m + 3)

(d) none

L1Difficulty1

Qtag Mathematics

Qcreator Pagemaker10

Q7. A father with 8 children takes them 3 at a time to the zoological garden, as often, he can without taking the same 3 children together more than once, the number of times each child will go to the garden is

(a) 56

(b) 21

(c) 112

(d) none

L1Difficulty1

Qtag Mathematics

Qcreator Pagemaker10

Q8. x, y, r are positive integers, then $x\_{Cr}+x\_{Cr-1}y\_{C1}+x\_{Cr-2}y\_{C2}$ + ............ + $y\_{Cr}$ is

(a) $\frac{x!y!}{r!}$

(b) $\frac{(x+y)!}{r!}$

(c) $x + y\_{Cr}$

(d) $xy\_{Cr}$

L1Difficulty1

Qtag Mathematics

Qcreator Pagemaker10

Q9. For 2 $\leq $ r $\leq $ n, $n\_{Cr}+2∙n\_{Cr-1}+n\_{Cr-2}$ + ............. is equal to

(a) $n+1\_{Cr-1}$

(b) $2∙n+1\_{Cr+1}$

(c) $2∙n+2\_{Cr}$

(d) $n+2\_{Cr}$

L1Difficulty1

Qtag Mathematics

Qcreator Pagemaker10

Q10. The number of positive integral solution of abc = 30 is

(a) 30

(b) 27

(c) 8

(d) 6

L1Difficulty1

Qtag Mathematics

Qcreator Pagemaker10

**Solutions**

S1. Ans. (a)

Sol.

1.3.5 (2n – 1) 2n

1.2.3. (2n – 1) 2n$∙$2n

= $\frac{2n! 2^{n}}{2^{n}\{1.2.3…….n\}}$ = $\frac{2n!}{n!}$

S2. Ans. (c)

Sol.

n! $\left\{1 – \frac{1}{1!} + \frac{1}{2!} + ............... (–1)^{n}.\frac{1}{n!}\right\}$

= 4! $\left\{\frac{1}{2!}- \frac{1}{3!} +\frac{1}{4!} \right\}$ = 12 – 4 + 1 = 9

S3. Ans. (c)

Sol.

$\left[\frac{100}{3}\right]+\left[\frac{100}{3^{2}}\right]+\left[\frac{100}{3^{3}}\right]+\left[\frac{100}{3^{4}}\right]+\left[\frac{100}{3^{5}}\right]$ + ....... 33 + 11 + 3 + 1 + 0 + ........

$$48$$

S4. Ans. (b)

S5. Ans. (a)

Sol.

First assign 2 marks to each question. Now remaining marks are 14 and they have to assign 8 question, so No. of ways 8 + 14 – $1\_{c8-1}$ = $21\_{c7}$

S6. Ans. (c)

Sol.

Required No.

Coefficient of x2m in (x0 + 11 + .......... xm)4

Coefficient of x2m in $\left(\frac{1-x^{m+1}}{1-x}\right)^{4}$

Coefficient of x2m in (1 – xm+1) (1 – x)–4

= $\frac{\left(m+1\right)(2m^{2}+4m+3)}{3}$

S7. Ans. (b)

Sol.

Each child will go as often as he (or she) can be accompanied by two others required No. is = $7\_{c2}=21$

S8. Ans. (c)

Sol.

The result is trivially true for r = 1, 2 it can be easily proved by the principal of mathematical induction that the result is true for r also.

S9. Ans. (d)

Sol.

$$n\_{Cr}+2∙n\_{Cr-1}+n\_{Cr-2}$$

$$n\_{Cr}+n\_{Cr-1}+n\_{Cr-1}+n\_{Cr-2}$$

$$n+1\_{Cr}+n+1\_{Cr-1}=n+2\_{Cr}$$

S10. Ans. (d)

Sol.

30 = 3 × 2 × 5 so a can take any value from 2, 3, 5 in 3 ways. ‘b’ can take the value in 2 ways and c in 1 ways.

No. of solution = 3 × 2 × 1 = 6

**LEVEL-II**

Q1. The number of ways in which the letters of the word ARRANGE can be arranged such that both R do not come together is

(a) 360

(b) 900

(c) 1260

(d) 1620

L3Difficulty3

Qtag Mathematics

Qcreator Pagemaker10

Q2. A box contains two white balls, three black balls and four red balls. In how many ways can three balls be drawn from the box if at least one black ball is to be included in the draw

(a) 64

(b) 45

(c) 46

(d) None of these

L3Difficulty3

Qtag Mathematics

Qcreator Pagemaker10

Q3. $m$ men and $n$ women are to be seated in a row so that no two women sit together. If $m>n,$ then the number of ways in which they can be seated as

(a) $\frac{m!\left(m+1\right)!}{\left(m-n+1\right)!}$

(b) $\frac{m!\left(m-1\right)!}{\left(m-n+1\right)!}$

(c) $\frac{\left(m-1\right)!\left(m+1\right)!}{\left(m-n+1\right)!}$

(d) None of these

L3Difficulty3

Qtag Mathematics

Qcreator Pagemaker10

Q4. A five digit number divisible by 3 has to be formed using the numbers 0, 1, 2, 3, 4 and 5 without repetition. The total number of ways in which this can be done is

(a) 216

(b) 240

(c) 600

(d) 3125

L3Difficulty3

Qtag Mathematics

Qcreator Pagemaker10

Q5. In a certain test there are $n$ questions. In the test $2^{n-1}$ students gave wrong answers to at least $i$ questions, where $i=1, 2,$ ………$n$. If the total number of wrong answers given is 2047, then $n$ is equal to

(a) 10

(b) 11

(c) 12

(d) 13

L3Difficulty3

Qtag Mathematics

Qcreator Pagemaker10

Q6. Total numbers of 4 digit-number which are not divisible by 5 are

(a) 7200

(b) 3600

(c) 14400

(d) 1800

L3Difficulty3

Qtag Mathematics

Qcreator Pagemaker10

Q7. Ten persons, amongst whom are $A, B$ and $C$ to speak at a function. The number of ways in which it can be done if $A$ wants to speak before $B$ and $B$ wants to speak before $C$ is

(a) $\frac{10!}{6}$

(b) $3!7!$

(c) 10*P3*$ .7!$

(d) None of these

L3Difficulty3

Qtag Mathematics

Qcreator Pagemaker10

Q8. If $$ then $n$ is equal to

(a) $1$

(b) $3$

(c) $5$

(d) 7

L3Difficulty3

Qtag Mathematics

Qcreator Pagemaker10

Q9. How many words can be made out from the letters of the word INDEPENDENCE, in which vowels always come together

(a) 16800

(b) 16630

(c) 1663200

(d) None of these

L3Difficulty3

Qtag Mathematics

Qcreator Pagemaker10

Q10. Five balls of different colours are to be placed in three boxes of different sizes. Each box can hold all five balls. In how many ways can we place the balls so that no box remains empty

(a) 50

(b) 100

(c) 150

(d) 200

L3Difficulty3

Qtag Mathematics

Qcreator Pagemaker10

**Solutions**

S1. Ans. (b)

Sol.

The word ARRANGE, has AA, RR, NGE letters, that is two A’s, two R’s and N, G, E one each.

$∴$ The total number of arrangements

$=\frac{7!}{2!2!1!1!1!}=1260$

But the number of arrangements in which both RR are together as one unit

$=\frac{6!}{2!1!1!1!1!}=360$

$∴$ The number of arrangements in which both R is do not come together $=1260-360=900.$

S2. Ans. (a)

Sol.

A selection of 3 balls so as to include at least one black ball, can be made in the following 3 mutually exclusive ways

(i) 1 black ball and 2 others = 3C1×6C2 = 3×15 = 45

(ii) 2 black balls and one other = 3C2×6C1 = 3×6 = 18

(iii) 3 black balls and no other = 3C3$=1$

$∴$ Total numbers of ways = $45+18+1=64.$

**Aliter :** Number of ways in which atleast one black ball can be taken

= Total ways – ways in which no black ball can be taken out

= 9C3 – 6C3= 84$-20=64.$

S3. Ans. (a)

Sol.

First arrange $m$ men, in a row in $m!$ ways. Since $n>m$ and no two women can sit together, in any one of the $m!$ arrangement, there are $(m+1)$ places in which $n$ women can be arranged in m+1Pn ways.

$∴$ By the fundamental theorem, the required number of arrangements of $m$ men and $n$ women $(n<m)$

= m!.m+1Pn = $\frac{m!.\left(m+1\right)!}{\left\{\left(m+1\right)-n\right\}!}=\frac{m!\left(m+1\right)!}{\left(m-n+1\right)!}$

S4. Ans. (a)

Sol.

We know that a five digit number is divisible by 3, and only if sum of its digits is divisible by 3, therefore we should not use 0 or 3 while forming the five digit numbers. Now, (1) In case we do not use 0 the five digit number can be formed (from the digit 1, 2 3, 4, 5) in 5P5 ways.

(ii) In case we do not use 3, the five digit number can be formed (from the digit 0, 1, 2, 4, 5) in 5P5 – 4P4 = 5 ! – 4 ! = 120 – 24 = 96 ways.

$∴$ The total number of such 5 digit number

= 5P5 + (5P5 – 4P4) = 120 + 96 = 216.

S5. Ans. (b)

Sol.

Since the number of students giving wrong answers in at least $i$ question $\left(i=1, 2, …….., n\right)=2^{n-1}.$

The number of students answering exactly $i(1\leq i\leq -3)$ questions wrongly = {the number of students answering at least $i$ questions wrong $i=1, 2, …….., n)\}-${the number of students answer at least $(i+1)$ questions wrongly $\left(2\leq i+1\leq r\right)=2^{n-1}-2^{n-\left(i+1\right)}\left(1\leq i\leq n-1\right).$

Now, the number of students answering all the questions wrongly = $2^{n-n}=2^{0}.$

Thus the total number of wrong answers

$$=1(2^{n-1}-2^{n-2}+2\left(2^{n-2}-2^{n-3}\right)+3\left(2^{n-3}+2^{n-4}\right)$$

 $+ $…………… $+\left(n-1\right)\left(2^{1}-2^{0}\right)+n(2)$

$=2^{n-1}+2^{n-2}+2^{n-3}+$ …………. $+2^{0}=2^{n}-1 (∵$ It’s a G.P.)

$∴$ As given $2^{n}-1=2047⇒2^{n}=2048=2^{11}⇒n=\frac{n}{2}$ .

S6. Ans. (a)

Sol.

The digit at the extreme right can be filled by 8 ways (as 0 and 5 have to be excluded). The digit at the extreme left can be filled by 9 ways and the next two digits can be each filled by 10 ways.

Hence total no. of ways are $9×10×10×8=7200.$

S7. Ans. (a)

Sol.

For A, B, C to speak in order of alphabets, 3 places one of 10 may be chosen first in 10C3 ways.

The remaining 7 persons can speak in 7! Ways. Hence, the number of ways in which all the 10 persons can speak is 10C3 . 7 !$ =\frac{10 !}{3 !} . =\frac{10 !}{6}$ .

S8. Ans. (d)

Sol.

$\frac{}{}=24⇒r!=24⇒r=4$

$∴ $

S9. Ans. (a)

Sol.

Required number of ways are $\frac{8 !}{2 ! 3 !}×\frac{5 !}{4 !}=16800.$

{Since IEEEENDPNDNC = 8 letters}.

S10. Ans. (c)

Sol.

Let the boxes be marked as $A, B, C. $We have to ensure that no box remains empty and in all five balls have to put in. There will be two possibilities.

(i) Any two containing one and 3rd containing 3.

$A\left(1\right) B\left(1\right) C(3)$

D5C1 . 4C1 . 3C3$= $5 . 4 . 1 = 20.

Since the box containing 3 balls could be any of the three boxes $A, B, C.$

Hence the required number is $=20×3=60.$

(ii) Any two containing 2 each and 3rd containing 1.

$A\left(2\right) B\left(2\right) C\left(1\right)$

5C2 . 3C2$. $1C1$= $10×3×1 = 30

Since the box containing 1 ball could be any of the three boxes $A, B, C.$

Hence the required number is $=30×3=90.$

Hence total number of ways are = $60+90=150.$

**LEVEL-III**

Q1. If $$ then $n=$

(a) 5

(b) 6

(c) 8

(d) 10

L5Difficulty5

Qtag Mathematics

Qcreator Pagemaker10

Q2. The total number of different combinations of one or more letters which can be made from the letters of the word ‘MISSISSIPPI’ is

(a) 150

(b) 148

(c) 149

(d) None of these

L5Difficulty5

Qtag Mathematics

Qcreator Pagemaker10

Q3. There were two women participating in a chess tournament. Every participant played two games with the other participants. The number of games that the men played between themselves proved to exceed by 66 the number of games that the men played with the women. The number of participants is

(a) 6

(b) 11

(c) 13

(d) None of these

L5Difficulty5

Qtag Mathematics

Qcreator Pagemaker10

Q4. In how many ways can Rs. 16 be divided into 4 person when none of them get less than Rs. 3

(a) 790

(b) 35

(c) 64

(d) 192

L5Difficulty5

Qtag Mathematics

Qcreator Pagemaker10

Q5. A dictionary is printed consisting of 7 lettered words only that can be made with a letter of the word CRICKET. If the words are printed at the alphabetical order, as in an ordinary dictionary, then the number of word before the word CRICKET is

(a) 530

(b) 480

(c) 531

(d) 481

L5Difficulty5

Qtag Mathematics

Qcreator Pagemaker10

Q6. A car will hold 2 in the front seat and 1 in the rear seat. If among 6 persons 2 can drive, then the number of ways in which the car can be filled is

(a) 10

(b) 20

(c) 30

(d) None of these

L5Difficulty5

Qtag Mathematics

Qcreator Pagemaker10

Q7. There are $(n+1)$ white and $(n+1)$ black balls each set numbered 1 to $n+1$. The number of ways in which the balls can be arranged in a row so that the adjacent balls are of different colours is

(a) $\left(2n+2\right)!$

(b) $\left(2n+2\right)!×2$

(c) $\left(n+1\right)!×2$

(d) $2\left\{\left(n+1\right)!\right\}^{2}$

L5Difficulty5

Qtag Mathematics

Qcreator Pagemaker10

Q8. 12 persons are to be arranged to a round table. If two particular persons among them are not to be side by side, the total number of arrangements is

(a) 9(10 !)

(b) 2(10 !)

(c) 45(8 !)

(d) 10 !

L5Difficulty5

Qtag Mathematics

Qcreator Pagemaker10

Q9. How many numbers between 5000 and 10,000 can be formed using the digits 1, 2, 3, 4, 5, 6, 7, 8, 9 each digit appearing not more than once in each number

(a) $5×^{8}P\_{3}$

(b) $5×^{8}C\_{3}$

(c) $5 !×^{8}P\_{3}$

(d) $5 !×^{8}C\_{3}$

L5Difficulty5

Qtag Mathematics

Qcreator Pagemaker10

Q10. A set contains $(2n+1)$ elements. The number of sub-sets of the set which contain at most $n$ elements is

(a) $2^{n}$

(b) $2^{n+1}$

(c) $2^{n-1}$

(d) $2^{2n}$

L5Difficulty5

Qtag Mathematics

Qcreator Pagemaker10

**Solutions**

S1. Ans. (a)

Sol.

$\frac{n!}{\left(n-3\right)!}+\frac{n!}{\left(n-2\right)! 2!}=14n $

$\left(n-1\right)\left(2n-3\right)=28⇒n=5,-5/2.$ So$, n=5.$

S2. Ans. (c)

Sol.

Here we have $1 M, 4 I, 4 S$ and $2P.$

Therefore total number of selections of one or more letters $=\left(1+1\right)\left(4+1\right)\left(4+1\right)\left(2+1\right)-1=149.$

S3. Ans. (c)

Sol.

Let there be $n$ men participants. Then the number of games that the men play between themselves is 2. *cC*2 and the number of games that the men played with the women is 2. $\left(2n\right).$

$∴$ 2. *nC*2–2. $2n=66$ (By hypothesis)

$⇒$ $n^{2}-5n-66=0⇒n=11$

$∴$ Number of participants = 11 men + 2 women = 13.

S4. Ans. (b)

Sol.

Required number of ways

= coefficient of $x^{16}$ in $\left(x^{3}+x^{4}+x^{5}+ …. +x^{7}\right)^{4}$

= coefficient of $x^{16}$ in $x^{12}\left(1+x+x^{2}+ ….x^{4}\right)^{4}$

= coefficient of $x^{16}$ in $x^{12}\left(1-x^{5}\right)^{4}\left(1-x\right)^{-4}$

= coefficient of $x^{4}$ in $\left(1-x^{5}\right)^{4}\left(1-x\right)^{-4}$

= coefficient of $x^{4}$ in $(1-4x^{5}+ …)$

 $\left[1+4x+ …+\frac{\left(r+1\right)\left(r+2\right)(r+3)}{3!}x^{r}\right] $

= $\frac{\left(4+1\right)\left(4+2\right)(4+3)}{3!}$=35

**Aliter :** Remaining 4 rupees can be distributed in $$ 35 ways.

S5. Ans. (a)

Sol.

The number of words before the word CRICKET is 4×5!+2×4!+2! = 530.

S6. Ans. (d)

Sol.

Since 2 persons can drive the car, therefore we have to select 1 from these two. This can be done in 2*C*1 ways. Now from the remaining 5 persons we have to select 2 which can be done in 5*C*2 ways. But the front seat and the rear seat person can interchange among themselves. Therefore, the required number of ways in which the car can be filled is 5*C*2×2*C*1×2! = 20×2 = 40.

S7. Ans. (d)

Sol.

Since the balls are to be arranged in a row so that the adjacent balls are of different colours, therefore we can begin with a white ball or a black ball. If we begin with a white ball, we find that $(n+1)$ white balls numbered 1 to $(n+1)$ can be arranged in a row in $\left(n+1\right)!$ ways. Now $(n+1)$ places are created between $n+1$ white balls which can be filled by $(n+1)$ black balls in $\left(n+1\right)!$ ways.

So the total number of arrangements in which adjacent balls are of different colours and first ball is a white ball is $\left(n+1\right)!×\left(n+1\right)!=\left[\left(n+1\right)!\right]^{2}.$ But we can begin with a black ball also. Hence the required number of arrangements is $2\left[\left(n+1\right)!\right]^{2}.$

S8. Ans. (a)

Sol.

12 persons can be seated around a round table in 11! Ways. The total number of ways in which 2 particular persons sit side by side is 10!×2!. Hence the required number of arrangements = $11!-10!×2!=9×10!.$

S9. Ans. (a)

Sol.

A number between 5000 and 10,000 can have any of the digits 5, 6, 7, 8, 9 at thousands place. So thousands place can be filled in 5 ways. Remaining places can be filled by the remaining 8 digits in ways. Hence required number = $5×$8*P*3 *.*

S10. Ans. (d)

Sol.

The number of sub-sets of the set which contain at most $n$ elements is

 $$…. $+$ (Say)

Then $2S=2($ ….. + $$

 = $($ ….

 …. $+($

 $\{∵$

 = $$ …... $+ $

$⇒S=2^{2n} $