**MATHEMATICS**

**PAGEMAKER10**

**ELLIPSE**

Q1. Distance between the directrix of the ellipse $\frac{x^{2}}{36}+\frac{y^{2}}{20}=1$ is

(a) 8

(b) 12

(c) 18

(d) 24

L1Difficulty1

Qtag Mathematics

Qcreator Pagemaker10

Q2. The distance between the foci of the ellipses 3x2 + 4y2 = 48 is

(a) 2

(b) 4

(c) 6

(d) 8

L1Difficulty1

Qtag Mathematics

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Q3. If the eccentricity of an ellipse be $\frac{1}{\sqrt{2}}$ , then its latus rectum is equal to

(a) minor axis

(b) semi-minor axis

(c) major-axis

(d) semi-major axis

L1Difficulty1

Qtag Mathematics

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Q4. The equation $\frac{x^{2}}{2-r}+\frac{y^{2}}{r-5}+1=0$ represent an ellipse, if

(a) r > 2

(b) 2 < r < 5

(c) r > 5

(d) none

L1Difficulty1

Qtag Mathematics

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Q5. The locus of the point which moves such that the ration of its distance from two lined point is the plane is always a constant k (< 1) is

(a) Hyperbola

(b) Ellipse

(c) Straight Line

(d) Circle

L1Difficulty1

Qtag Mathematics

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Q6. If the line y = 2x + c be a tangent to the ellipse $\frac{x^{2}}{8}+\frac{y^{2}}{4}=1$ then c is

(a) ± 4

(b) ± 6

(c) ± 1

(d) ± 8

L1Difficulty1

Qtag Mathematics

Qcreator Pagemaker10

Q7. Minimum area of the triangle by any tangent to the ellipse $\frac{x^{2}}{a^{2}}+\frac{y^{2}}{b^{2}}=1$ with the co-ordinate axes is

(a) $\frac{a^{2}+b^{2}}{2}$

(b) $\frac{(a+b)^{2}}{2}$

(c) ab

(d) $\frac{(a-b)^{2}}{2}$

L1Difficulty1

Qtag Mathematics

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Q8. If the line x Cos $α$ + y Sin $α$ = p be normal to the ellipse $\frac{x^{2}}{a^{2}}+\frac{y^{2}}{b^{2}}=1$, then

(a) p2 (a2Cos2$α$+ b2 Sin2$α$) = a2 – b2

(b) p2 (a2 Cos2$α$ + b2 Sin2$α$) = (a2 – b2)2

(c) p2 (a2 + Sec2$α$ + b2 Cosec2$α$) = a2 – b2

(d) p2 (a2 Sec2$α$+ b2 Cosec2$α$) = (a2 – b2)2

L1Difficulty1

Qtag Mathematics

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Q9. An ellipse has OB as a semi minor axis F, F' are its foci and the angle FBF' at a right angle. The eccentricity of the ellipse is

(a) $\frac{1}{\sqrt{2}}$

(b) $\frac{1}{2}$

(c) $\frac{1}{3}$

(d) $\frac{1}{4}$

L1Difficulty1

Qtag Mathematics

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Q10. The ellipse E1 : $\frac{x^{2}}{9}+ \frac{y^{2}}{4}=1$ is inscribed in a rectangle R whose lines are parallel to the co-ordinate axis. Another ellipse E2 passing through the point (0, 4) circumscribes the rectangle R. The eccentricity of the ellipse E2 is

(a) $\frac{\sqrt{2}}{2}$

(b) $\frac{\sqrt{3}}{2}$

(c) $\frac{1}{2}$

(d) $\frac{3}{4}$

L1Difficulty1

Qtag Mathematics

Qcreator Pagemaker10

**Solutions**

S1. Ans. (c)

Sol.

a = 6, b = 2$\sqrt{5}$

b2 = a2(1 – e2)

Directrix are x = ± $\frac{a}{e}$

Distance between them $2∙\frac{6}{2/3}$

18.

S2. Ans. (b)

Sol.

a2 = 16, b2 = 12, e = $\sqrt{1-\frac{b^{2}}{a^{2}}}$

Distance is 2ae = 4.

S3. Ans. (d)

Sol.

e = $\frac{1}{\sqrt{2}}$

L.R. = $\frac{2b^{2}}{a}$ = $\frac{2a^{2}}{a}$ $\left(1-\frac{1}{2}\right)$ = a

semi major axis

S4. Ans. (b)

Sol.

$$\frac{x^{2}}{r-2}+\frac{y^{2}}{5-r}=1$$

r > 2 and r < 5

S5. Ans. (b)

Sol.

According to the definition.

S6. Ans. (b)

Sol.

c = $\sqrt{a^{2}m^{2}+b^{2}}$

c = $\sqrt{8×4+4}$

c = ± 6

S7. Ans. (c)

Sol.



$\frac{x Cos θ}{a}+\frac{y Sin θ}{b}=1$

Area $\frac{1}{2}$ a sec $θ.b$ Cosec $θ$

$\frac{1}{2}$ × $\frac{2ab}{2 Sinθ.Cosθ}$ = $\frac{ab}{Sin2θ}$ > ab

S8. Ans. (a)

Sol.

Normal is ax sec $ϕ$ – by Cosec $ϕ$ = a2 – b2 straight line x Cos $α$ + y Sin$ α$ = p will be normal to the ellipse $\frac{x^{2}}{a^{2}}+\frac{y^{2}}{b^{2}}=1$

$\frac{a Secϕ}{Cosα}=\frac{-b Cosecϕ}{Sinα}=\frac{a^{2}-b^{2}}{p}$

Cos $ϕ$ = $\frac{ap}{\left(a^{2}-b^{2}\right)Cosα}$

Sin $ϕ$ = $\frac{-bp}{\left(a^{2}-b^{2}\right)Sinα}$

p2 (b2 Cosec2$α$ + a2sec2$α$) = (a2 – b2)2

S9. Ans. (a)

Sol.



Slope of BF is $\frac{b-O}{O-ae}$

Slope of BF' is $\frac{b-O}{O+ae}$

$∠$F BF' = 90 $\frac{b}{-ae}×\frac{b}{ae}= –1$

$$e=\frac{1}{\sqrt{2}}$$

S10. Ans. (c)



Sol.

(y + 2) (y – 2) + $λ$ (x + 3) (x – 3) = 0

it passes through (0, 4) $λ$ = $\frac{4}{3}$

$\frac{x^{2}}{12}+\frac{y^{2}}{16}=1$

e = $\frac{1}{2}$

**LEVEL-II**

Q1. If $2y=x$ and $3y+4x=0$ are the equations of a pair of conjugate diameters of an ellipse, then the eccentricity of the ellipse is

(a) $\sqrt{\frac{2}{3}}$

(b) $\sqrt{\frac{2}{5}}$

(c) $\sqrt{\frac{1}{3}}$

(d) $\sqrt{\frac{1}{2}}$

L3Difficulty3

Qtag Mathematics

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Q2. The distance of the point $‘θ’$ on the ellipse $\frac{x^{2}}{a^{2}}+\frac{y^{2}}{b^{2}}=1$ from a focus is

(a) $a(e+coscos θ) $

(b) $a(e-coscos θ) $

(c) $a(1+ecoscos θ) $

(d) $a(1+2ecoscos θ) $

L3Difficulty3

Qtag Mathematics

Qcreator Pagemaker10

Q3. The equation of the ellipse whose one focus is at (4, 0) and whose eccentricity is 4/5, is

(a) $\frac{x^{2}}{3^{2}}+\frac{y^{2}}{5^{2}}=1$

(b) $\frac{x^{2}}{5^{2}}+\frac{y^{2}}{3^{2}}=1$

(c) $\frac{x^{2}}{5^{2}}+\frac{y^{2}}{4^{2}}=1$

(d) $\frac{x^{2}}{4^{2}}+\frac{y^{2}}{5^{2}}=1$

L3Difficulty3

Qtag Mathematics

Qcreator Pagemaker10

Q4. The foci of $16x^{2}+25y^{2}=400$ are

(a) $(\pm 3, 0)$

(b) $(0, \pm 3)$

(c) $(3, -3)$

(d) $(-3, 3)$

L3Difficulty3

Qtag Mathematics

Qcreator Pagemaker10

Q5. The line passing through the extremity $A$ of the major axis and extremity $B$ of the minor axis of the ellipse $x^{2}+9y^{2}=9$ meets its auxiliary circle at the point $M$. Then the area of the triangle with vertices at $A, M$ and the origin $O$ is

(a) $\frac{31}{10}$

(b) $\frac{29}{10}$

(c) $\frac{21}{10}$

(d) $\frac{27}{10}$

L3Difficulty3

Qtag Mathematics

Qcreator Pagemaker10

Q6. The normal at a point $P$ on the ellipse $x^{2}+4y^{2}=16$ meets the $x$-axis at $Q$. If $M$ is the mid point of the line segment $PQ$, then the locus of $M$ intersects the latus rectums of the given ellipse at the points

(a) $\left(\pm \frac{3\sqrt{5}}{2},\pm \frac{2}{7}\right)$

(b) $\left(\pm \frac{3\sqrt{5}}{2},\pm \frac{\sqrt{19}}{4}\right)$

(c) $\left(\pm 2\sqrt{3},\pm \frac{1}{7}\right)$

(d) $\left(\pm 2\sqrt{3},\pm \frac{4\sqrt{3}}{7}\right)$

L3Difficulty3

Qtag Mathematics

Qcreator Pagemaker10

Q7. The locus of a variable point whose distance from $(-2, 0)$ is $\frac{2}{3}$ times its distance from the line $x=-\frac{9}{2}$ , is

(a) Ellipse

(b) Parabola

(c) Hyperbola

(d) None of these

L3Difficulty3

Qtag Mathematics

Qcreator Pagemaker10

Q8. The equation of the tangent to the ellipse $x^{2}+16y^{2}=16$ making an angle of $60°$ with $x$-axis is

(a) $\sqrt{3}x-y+7=0$

(b) $\sqrt{3}x-y-7=0$

(c) $\sqrt{3}x-y\pm 7=0$

(d) None of these

L3Difficulty3

Qtag Mathematics

Qcreator Pagemaker10

Q9. The position of the point (1, 3) with respect to the ellipse $4x^{2}+9y^{2}-16x-54y+61=0$

(a) Outside the ellipse

(b) On the ellipse

(c) On the major axis

(d) On the minor axis

L3Difficulty3

Qtag Mathematics

Qcreator Pagemaker10

Q10. The line $lx+my-n=0$ will be tangent to the ellipse $\frac{x^{2}}{a^{2}}+\frac{y^{2}}{b^{2}}=1,$ if

(a) $a^{2}l^{2}+b^{2}m^{2}=n^{2}$

(b) $al^{2}+bm^{2}=n^{2}$

(c) $a^{2}l+b^{2}m=n$

(d) None of these

L3Difficulty3

Qtag Mathematics

Qcreator Pagemaker10

**Solutions**

S1. Ans. (c)

Sol.

$2y=x;3y+4x=0$ [Conjugate diameters]

$∴y=\frac{1}{2}x; y=\frac{-4}{3}x$

$∴\frac{-b^{2}}{a^{2}}=m\_{1}m\_{2}⇒\frac{-b^{2}}{a^{2}}=\frac{1}{2}\left(\frac{-4}{3}\right)=\frac{-2}{3}$

$∴e=\sqrt{1-\frac{b^{2}}{a^{2}}}=\sqrt{1-\frac{2}{3}}=\sqrt{\frac{1}{3}} .$

S2. Ans. (c)

Sol.

Focal distance of any point $P(x, y)$ on the ellipse is equal to $SP=a+ex.$ Here $x=acoscos θ $

Here, $SP=a+aecoscos θ=a(1+ecoscos θ). $

S3. Ans. (b)

Sol.

Here, $ae=4$ and $e=\frac{4}{5}⇒a=5$

Now $b^{2}=a^{2}\left(1-e^{2}\right)⇒b^{2}=25\left(1-\frac{16}{25}\right)=9$

Hence, equation of the ellipse is $\frac{x^{2}}{25}+\frac{y^{2}}{9}=1$.

S4. Ans. (a)

Sol.

The equation of the ellipse is $16x^{2}+25y^{2}=400$ or $\frac{x^{2}}{25}+\frac{y^{2}}{16}=1.$

Here, $a^{2}=25, b^{2}=16⇒e=\frac{3}{5} .$

S5. Ans. (d)

Sol.

Equation of auxiliary circle is $x^{2}+y^{2}=9$ ….(i)

Equation of $AM$ is $\frac{x}{3}+\frac{y}{1}=1$ ….(ii)

On solving (i) and (ii), we get $M\left(-\frac{12}{5},\frac{9}{5}\right)$

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Now, area of $∆AOM=\frac{1}{2} .OA×MN=\frac{27}{10}$ $square unit.$

S6. Ans. (c)

Sol.

$\frac{x^{2}}{16}+\frac{y^{2}}{4}=1, a=4, b=2$

Equation of normal $4xsecsec θ-2y cosec θ=12$

$M\left(\frac{7coscos θ }{2},sinsin θ \right)=(h, k)$ (say)

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$h=\frac{7coscos θ }{2}$

$⇒cosθ=\frac{2h}{7}$

and $k=sinsin θ $

$\frac{4h^{2}}{49}+k^{2}=1$

Locus $\frac{4x^{2}}{49}+y^{2}=1$ ….(i)

For given ellipse $e^{2}=1-\frac{4}{16}=\frac{3}{4}⇒e=\frac{\sqrt{3}}{2}$

$x=\pm 4×\frac{\sqrt{3}}{2}=\pm 2\sqrt{3} $ ….(ii)

Solving (i) and (ii), we get

$\frac{4}{49}×12+y^{2}=1⇒y^{2}=1-\frac{48}{49}⇒y=\pm \frac{1}{7}$

$∴ $Required points $\left(\pm 2\sqrt{3},\pm \frac{1}{7}\right).$

S7. Ans. (a)

Sol.

Let point $P\left(x\_{1}, y\_{1}\right)$

So, $\sqrt{\left(x\_{1}+2\right)^{2}+y\_{1}^{2}}=\frac{2}{3}\left(x\_{1}+\frac{9}{2}\right)$

$⇒\left(x\_{1}+2\right)^{2}+y\_{1}^{2}=\frac{4}{9}\left(x\_{1}+\frac{9}{2}\right)^{2}$

$⇒9\left[x\_{1}^{2}+y\_{1}^{2}+4x\_{1}+4\right]=4\left(x\_{1}^{2}+\frac{81}{4}+9x\_{1}\right) $

$⇒5x\_{1}^{2}+9y\_{1}^{2}=45⇒\frac{x\_{1}^{2}}{9}+\frac{y\_{1}^{2}}{5}=1,$

Locus of $(x\_{1}, y\_{1})$ is $\frac{x^{2}}{9}+\frac{y^{2}}{5}=1,$ which is equation of an ellipse.

**Trick :** The ratio $e=\frac{2}{3}<1.$ So, it is ellipse (fundamental concept).

S8. Ans. (c)

Sol.

$m=tantan 60°=\sqrt{3}. $ Therefore, equation of tangent is $y=\sqrt{3}x\pm \sqrt{1+3.16}⇒y=\sqrt{3}x\pm 7.$

S9. Ans. (c)

Sol.

$E≡4+9\left(3\right)^{2}-16\left(1\right)-54\left(3\right)+61<0$

Therefore, the point is inside the ellipse.

$\frac{4\left(x-2\right)^{2}}{36}+\frac{9\left(y-3\right)^{2}}{36}=1$

Equation of major axis is $y-3=0$ and point (1, 3) lies on it.

S10. Ans. (a)

Sol.

$y=\frac{-1}{m}x+\frac{n}{m}$ is tangent to $\frac{x^{2}}{a^{2}}+\frac{y^{2}}{b^{2}}=1,$ if

$\frac{n}{m}=\pm \sqrt{b^{2}+a^{2}\left(\frac{1}{m}\right)^{2}}$ or $n^{2}=m^{2}b^{2}+l^{2}a^{2}.$

**LEVEL-III**

Q1. The locus of the point of intersection of mutually perpendicular tangent to the ellipse $\frac{x^{2}}{a^{2}}+\frac{y^{2}}{b^{2}}=1,$ is

(a) A straight line

(b) A parabola

(c) A circle

(d) None of these

L5Difficulty5

Qtag Mathematics

Qcreator Pagemaker10

Q2. The equation of the tangent at the point $(1/4, 1/4 )$ of the ellipse $\frac{x^{2}}{4}+\frac{y^{2}}{12}=1$ is

(a) $3x+y=48$

(b) $3x+y=3$

(c) $3x+y=16$

(d) None of these

L5Difficulty5

Qtag Mathematics

Qcreator Pagemaker10

Q3. The angle between the pair of tangents drawn to the ellipse $3x^{2}+2y^{2}=5$ from the point (1, 2) is

(a) $\left(\frac{12}{5}\right) $

(b) $(6\sqrt{5}) $

(c) $\left(\frac{12}{\sqrt{5}}\right) $

(d) $(12\sqrt{5}) $

L5Difficulty5

Qtag Mathematics

Qcreator Pagemaker10

Q4. The equations of the tangents of the ellipse $9x^{2}+16y^{2}=144$ which passes through the point (2, 3) is

(a) $y=3, x+y=5$

(b) $y=-3, x-y=5$

(c) $y=4, x+y=3$

(d) $y=-4, x-y=3$

L5Difficulty5

Qtag Mathematics

Qcreator Pagemaker10

Q5. If any tangent to the ellipse $\frac{x^{2}}{a^{2}}+\frac{y^{2}}{b^{2}}=1$ cuts off intercepts of length $h$ and $k$ on the axes, then $\frac{a^{2}}{h^{2}}+\frac{b^{2}}{k^{2}}=$

(a) 0

(b) 1

(c) $-1$

(d) None of these

L5Difficulty5

Qtag Mathematics

Qcreator Pagemaker10

Q6. The value of $λ,$ for which the line $2x-\frac{8}{3}λy=-3$ is a normal to the conic $x^{2}+\frac{y^{2}}{4}=1$ is

(a) $\frac{\sqrt{3}}{2}$

(b) $\frac{1}{2}$

(c) $-\frac{\sqrt{3}}{2}$

(d) $\frac{3}{8}$

L5Difficulty5

Qtag Mathematics

Qcreator Pagemaker10

Q7. The pole of the straight line $x+4y=4$ with respect to ellipse $x^{2}+4y^{2}=4$ is

(a) $(1, 4)$

(b) $(1, 1)$

(c) $(4, 1)$

(d) $(4, 4)$

L5Difficulty5

Qtag Mathematics

Qcreator Pagemaker10

Q8. The sum of focal distances of any point on the ellipse with major and minor axes as $2a$ and $2b$ respectively, is equal to

(a) $2a$

(b) $\frac{2a}{b}$

(c) $\frac{2b}{a}$

(d) $\frac{b^{2}}{a}$

L5Difficulty5

Qtag Mathematics

Qcreator Pagemaker10

Q9. The equation of ellipse whose distance between the foci is equal to 8 and distance between the directrix is 18, is

(a) $5x^{2}-9y^{2}=180$

(b) $9x^{2}+5y^{2}=180$

(c) $x^{2}+9y^{2}=180$

(d) $5x^{2}+9y^{2}=180$

L5Difficulty5

Qtag Mathematics

Qcreator Pagemaker10

Q10. In an ellipse the distance between its foci is 6 and its minor axis is 8. Then its eccentricity is

(a) $\frac{4}{5}$

(b) $\frac{1}{\sqrt{52}}$

(c) $\frac{3}{5}$

(d) $\frac{1}{2}$

L5Difficulty5

Qtag Mathematics

Qcreator Pagemaker10

**Solutions**

S1. Ans. (c)

Sol.

It is a fundamental concept.

S2. Ans. (d)

Sol.

The point does not lie on ellipse.

S3. Ans. (c)

Sol.

$SS\_{1}=T^{2}$

$tantan θ=2\frac{\sqrt{h^{2}-ab}}{a+b},a=9, b=-4 $ and $h=-12.$

S4. Ans. (a)

Sol.

The tangent will be $y-3=m\left(x-2\right)⇒y-mx=3-2m.$ But it is tangent to the given ellipse, therefore $m=0, -1.$ Hence tangents are $y=3$ and $x+y=5.$

**Trick :** Only answer (a) satisfy the point (2, 3).

S5. Ans. (b)

Sol.

The tangent at $(acoscos θ, bsinsin θ) $ to the ellipse is

$\frac{(acoscos θ)x }{a^{2}}+\frac{(bsinsin θ)y }{b^{2}}=1$ or $\frac{x}{(a/coscos θ) }+\frac{y}{(b/sinsin θ) }=1$

$∴$ Intercepts are, $h=\frac{a}{coscos θ }, k=\frac{b}{sinsin θ }⇒\frac{a^{2}}{h^{2}}+\frac{b^{2}}{k^{2}}=1. $

S6. Ans. (d)

Sol.

We know that the equation of the normal at point $(acoscos θ, bsinsin θ) $ on the curve $x^{2}+\frac{y^{2}}{4}=1$ is given by

$axsecsec θ-by cosec θ=a^{2}-b^{2} $ ….(i)

Comparing equation (i) with $2x-\frac{8}{3}λy=-3,$ we get

$asecsec θ=2, b cosec θ=\frac{8}{3}λ $ or $ab=\frac{16}{3}λ$ ….(ii)

$∴a=1, b=2;$

$∴2=\frac{16}{3}λ$ or $λ=3/8$

S7. Ans. (b)

Sol.

We know that equation of polar at point $(h, k)$ is

$\frac{hx}{a^{2}}+\frac{ky}{b^{2}}=1⇒\frac{hx}{4}+\frac{ky}{1}=1⇒hx+4ky=4$ ….(i)

Which is similar to given straight line $x+4y=4$ ….(ii)

Comparing (i) and (ii), we get $h=1, k=1.$

Hence the point is $\left(1, 1\right).$

S8. Ans. (a)

Sol.

Sum of focal distances of a point in an ellipse is always equal to length of major axis of that ellipse. It is a property of ellipse.

S9. Ans. (d)

Sol.

$2ae=8,\frac{2a}{e}=18⇒a=\sqrt{4×9}=6 $

$e=\frac{2}{3}, b=6\sqrt{1-\frac{4}{9}}=\frac{6}{3}\sqrt{5}=2\sqrt{5}$

Hence, the required equation is $\frac{x^{2}}{36}+\frac{y^{2}}{20}=1$

$i.e.,$ $5x^{2}+9y^{2}=180.$

S10. Ans. (c)

Sol.

Distance between foci = 6$ ⇒ae=3.$

Minor axis = 8$ ⇒2b=8⇒b=4⇒b^{2}=16$

$⇒a^{2}\left(1-e^{2}\right)=16⇒a^{2}-a^{2}e^{2}=16⇒a^{2}-9=16⇒a=5$

Hence, $ae=3⇒e=\frac{3}{5} .$