UPSC CDS 1 2021 Elementary Mathematics Previous Year Paper : 10th Nov

Q1. $x^3 + x^2 + 16$, is exactly divisible by x, where x is a positive integer. The number of all such possible values of x is

(a) 3 (b) 4
(c) 5 (d) 6

Q2. The number of (a, b, c), where a, b, c are positive integers such that abc = 30, is

(a) 30 (b) 27
(c) 9 (d) 8

Q3. If the roots of the quadratic equation $x^2 - 4x - \log_{10} N = 0$ are real, then what is the minimum value of N ?

(a) 1 (b) $\frac{1}{10}$
(c) $\frac{1}{100}$ (d) $\frac{1}{10000}$

Q4. An article is sold for Rs. 2691 after successive discount of 8% and 22%. What is the marked price of the article?

(a) Rs. 4250 (b) Rs. 3750
(c) Rs. 4550 (d) RS. 3200

Q5. If $I = a^2 + b^2 + c^2$, where a and b are consecutive integers and c = ab, then I is

(a) an even number and it is not a square of an integer
(b) an odd number and it is not a square of an integer
(c) square of an even integer
(d) square of an odd integer

Q6. If the number 23P62971335 is divisible by the smallest odd composite number, then what is the value of P?

(a) 4 (b) 5
(c) 6 (d) 7

Q7. What is the remainder when the sum $1^5 + 2^5 + 3^5 + 4^5 + 5^5$ is divided by 4 ?

(a) 0 (b) 1
(c) 2 (d) 3

Q8. What is the digit in the unit place of 3^{99} ?

(a) 1 (b) 3
(c) 7 (d) 9

Q9. LCM of two numbers is 28 times their HCF. The sum of the HCF and the LCM is 1740. If one of these numbers is 240, then what is the other number?

(a) 420 (b) 640
(c) 820 (d) 1040

Q10. $(x^n - a^n)$ is divisible by $(x - a)$, where $x \neq a$, for every

(a) natural number n
(b) even natural number n only
(c) odd natural number n only
(d) prime number n only

Q11. If 17^{2020} is divided by 18, then what is the remainder?

(a) 1 (b) 2
(c) 16 (d) 17

Q12. What is the value of

$$\frac{1}{1+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{4}} + \ldots + \frac{1}{\sqrt{99}+\sqrt{100}} ?$$

(a) 1 (b) 5
(c) 9 (d) 10

Q13. If $x^m = \sqrt[14]{x\sqrt{x\sqrt{x}}}$, then what is the value of m ?

(a) $\frac{1}{8}$ (b) $\frac{1}{4}$
(c) $\frac{3}{4}$ (d) $\frac{7}{4}$

Q14. The sum of all possible products taken two at a time out of the numbers ± 1, ±2, ± 3, ±4, ±5 is
(a) 0 (b) –30
(c) –55 (d) 55

Q15. A train of length 110 m is moving at a uniform speed of 132 km/hr. The time required to cross a bridge of length 165 m is
(a) 6:5 seconds (b) 7 seconds
(c) 7.5 seconds (d) 8.5 seconds

Q16. The simple interest on a certain sum is one-fourth of the sum. If the number of years and the rate of annual interest are numerically equal, then the number of years is
(a) 2.5 (b) 3
(c) 3.5 (d) 5

Q17. A 60-page book has n lines per page. If the number of lines were reduced by 3 in each page, the number of pages would have to be increased by 10 to give the same writing space. What is the value of n ?
(a) 18 (b) 21
(c) 24 (d) 30

Q18. If x men working x hours per day can do x units of work in x days, then y men working y hours per day in y days would be able to do k units of work. What is the value of k ?
(a) x^2y^{-3} (b) x^3y^{-2}
(c) y^2x^{-3} (d) y^3x^{-2}

Q19. Let d(n) denote the number of positive divisors of a positive integer n. Which of the following are correct ?
1. d(5) = d(11)
2. d(5) d(11) = d(55)
3. d(5) + d(11) = d(16)
Select the correct answer using the code given below :
(a) 1 and 3 only (b) 1 and 2 only
(c) 2 and 3 only (d) 1, 2 and 3

Q20. If $A_n = P_n + 1$, where P_n is the product of the first n prime numbers, then consider the following statements :
1. A_n is always a composite number.
2. $A_n + 2$ is always an odd number.
3. $A_n + 1$ is always an even number.
Which of the above statements is/are correct?
(a) 1 only (b) 2 only
(c) 3 only (d) 2 and 3 only

Q21. A shopkeeper sells his articles at their cost price but uses a faulty balance which reads 1000 gm for 800 gm. What is the actual profit percentage ?
(a) 20% (b) 25%
(c) 30% (d) 40%

Q22. A river 3 m deep and 40 m wide is flowing at the rate of 2 km/hr and falls into the sea. What is the amount of water in litres that will fall into the sea from this river in a minute ?
(a) 40,00,000 litres (b) 4,00,000 litres
(c) 40,000 litres (d) 4,000 litres

Q23. If a television set is sold at Rs x, a loss of 28% would be incurred. If it is sold at Rs y, a profit of 12% would be incurred. What is the ratio of y to x?
(a) 41 : 9 (b) 31 : 9
(c) 23 : 9 (d) 14 : 9

Q24. By increasing the speed of his car by 15 km/hr, a person covers a distance of 300 km by taking an hour less than before. What was the original speed of the car ?
(a) 45 km/hr (b) 50 km/hr
(c) 60 km/hr (d) 75 km/hr

Q25. Three persons start a business with capitals in the ratio $\frac{1}{3} : \frac{1}{4} : \frac{1}{5}$. The first person withdraws half his capital after 4 months. What is his share of profit if the business fetches an annual profit of Rs 96,800 ?
(a) Rs 32,000 (b) Rs 34,500
(c) Rs 36,000 (d) Rs 36,800

Q26. If x varies as y, then which of the following is/are correct ?
1. $x^2 + y^2$ varies as $x^2 - y^2$
2. $\frac{x}{y^2}$ varies inversely as y
3. $\sqrt[n]{x^2y}$ varies as $\sqrt[2n]{x^4y^2}$
Select the correct answer using the code given below :
(a) 1 and 2 only (b) 2 and 3 only
(c) 3 only (d) 1, 2 and 3

Q27. Ena was born 4 years after her parents marriage. Her mother is 3 years younger than her father and 24 years older than Ena, who is 13 years old. At what age did Ena's father get married ?
(a) 25 years (b) 24 years
(c) 23 years (d) 22 years

Q28. Mahesh is 60 years old. Ram is 5 years younger to Mahesh and 4 years elder to Raju. Babu is a younger brother of Raju and he is 6 years younger. What is the age difference between Mahesh and Babu ?
(a) 18 years (b) 15 years
(c) 13 years (d) 11 years

Q29. The number of items in a booklet is N. In the first year there is an increase of x% in this number and in the subsequent year there is a decrease of x%. At the end of the two years, what will be the number of items in the booklet?
(a) Less than N
(b) Equal to N
(c) More than N
(d) It depends on the value of N

Q30. If ab + xy – xb = 0 and bc + yz – cy = 0, then what is $\frac{x}{a}+\frac{c}{z}$ equal to?
(a) $\frac{y}{b}$ (b) $\frac{b}{y}$
(c) 1 (d) 0

Q31. What is the HCF of the polynomials
$x^6 - 3x^4 + 3x^2 - 1$ and $x^3 + 3x^2 + 3x + 1$?
(a) (x + 1) (b) $(x + 1)^2$
(c) $x^2 + 1$ (d) $(x + 1)^3$

Q32. The HCF and the LCM of two polynomials are 3x + 1 and $30x^3 + 7x^2 - 10x - 3$ respectively. If one polynomial is $6x^2 + 5x + 1$, then what is the other polynomial ?
(a) $15x^2 + 4x + 3$ (b) $15x^2 + 4x - 3$
(c) $15x^2 - 4x + 3$ (d) $15x^2 - 4x - 3$

Q33. If (p + 2) (2q – 1) = 2pq – 10 and (p – 2) (2q – 1) = 2pq – 10, then what is pq equal to ?
(a) –10 (b) –5
(c) 5 (d) 10

Q34. What is the value of
$$\frac{a^2+ac}{a^2c-c^3}-\frac{a^2-c^2}{a^2c+2ac^2+c^3}-\frac{2c}{a^2-c^2}+\frac{3}{a+c}?$$
(a) 0 (b) 1
(c) $\frac{ac}{a^2+c^2}$ (d) $\frac{6}{a+c}$

Q35. What is the square root of $4x^4 + 8x^3 - 4x + 1$?
(a) $2x^2 - 2x - 1$ (b) $2x^2 - x - 1$
(c) $2x^2 + 2x + 1$ (d) $2x^2 + 2x - 1$

Q36. The sum of the digits of a two digit number is 13 and the difference between the number and that formed by reversing the digits is 27. What is the product of the digits of the number?
(a) 35 (b) 40
(c) 45 (d) 54

Q37. If $\frac{x}{b+c}=\frac{y}{c+a}=\frac{z}{b-a}$, then which one of the following is correct ?
(a) x + y + z = 0 (b) x – y – z = 0
(c) x + y – z = 0 (d) x + 2y + 3z = 0

Q38. X, Y and Z travel from the same place with uniform speeds 4 km/hr, 5 km/hr and 6 km/hr respectively. Y starts 2 hours after X. How long after Y must Z start in order that they overtake X at the same instant ?
(a) $\frac{3}{2}$ hours (b) $\frac{4}{3}$ hours
(c) $\frac{9}{8}$ hours (d) $\frac{11}{8}$ hours

Q39. $1 - x - x^n + x^{n+1}$, where n is a natural number, is divisible by
(a) $(1 + x)^2$ (b) $(1 - x)^2$
(c) $1 - 2x - x^2$ (d) $1 + 2x - x^2$

Q40. A person sold an article for Rs 75 which cost him Rs x. He finds that he realised x% profit on his outlay. What is x equal to ?
(a) 20% (b) 25%
(c) 50% (d) 100%

Q41. A car did a journey in t hours. Had the average speed been x kmph greater, the journey would have taken y hours less. How long was the journey ?
(a) x(t – y) ty (b) $x(t - y)\, ty^{-1}$
(c) $x(t - y)\, t\, y^{-2}$ (d) x(t + y) ty

Q42. When a ball is allowed to fall, the time it takes to fall any distance varies as the square root of the distance and it takes 4 seconds to fall 78.40 m. How long would it take to fall 122.50 m ?
(a) 5 seconds (b) 5.5 seconds
(c) 6 seconds (d) 6.5 seconds

Q43. If $6^{3-4x} 4^{x+5} = 8$ (Given $\log_{10} 2 = 0.301$ and $\log_{10} 3 = 0.477$), then which one of the following is correct ?
(a) 0 < x < 1 (b) 1 < x < 2
(c) 2 < x < 3 (d) 3 < x < 4

Q44. The Euclidean algorithm is used to calculate the
(a) square root of an integer
(b) cube root of an integer
(c) square of an integer
(d) HCF of two integers

Q45. If radius of a sphere is rational, then which of the following is/are correct ?
1. Its surface area is rational.
2. Its volume is rational.
Select the correct answer using the code given below :
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

Q46. If cosec θ – sin θ = m and sec θ – cos θ = n, then what is $m^{\frac{4}{3}}n^{\frac{2}{3}} + m^{\frac{2}{3}}n^{\frac{4}{3}}$ equal to ?
(a) 0 (b) 1
(c) mn (d) m^2n^2

Q47. If cos θ + sec θ = k, then what is the value of $\sin^2\theta - \tan^2\theta$?
(a) 4 – k (b) $4 - k^2$
(c) $k^2 - 4$ (d) $k^2 + 2$

Q48. ABC is a triangle inscribed in a semicircle of diameter AB. What is cos (A + B) + sin (A + B) equal to ?
(a) 0 (b) $\frac{1}{4}$
(c) $\frac{1}{2}$ (d) 1

Q49. Δ XYZ ~Δ TUS and XY = 6, YZ = 10 and ZX = 9. If ar(Δ XYZ) : ar(Δ TUS) = 25:16, then US is equal to :
(a) 8 cm (b) 10 cm
(c) 7.5 cm (d) 6 cm

Q50. What is the magnitude (in radian) of the interior angle of a regular pentagon ?
(a) $\frac{\pi}{5}$ (b) $\frac{2\pi}{5}$
(c) $\frac{3\pi}{5}$ (d) $\frac{4\pi}{5}$

Q51. The difference between two angles is 15° and the sum of the angles in radian is $\frac{5\pi}{12}$. The bigger angle is k times the smaller angle. What is k equal to ?
(a) $\frac{4}{3}$ (b) $\frac{3}{2}$
(c) $\frac{6}{5}$ (d) $\frac{7}{6}$

Q52. Consider the following statements :
1. The equation $2\sin^2\theta - \cos\theta + 4 = 0$ is possible for all θ.
2. tan θ + cot θ cannot be less than 2, where $0 < \theta < \frac{\pi}{2}$.
Which of the above statements is/are correct ?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

Q53. A road curve is to be laid out on a circle. What radius should be used if the track is to change direction by 42° in distance of 44 m ?
$\left(\text{Assume } \pi = \frac{22}{7}\right)$
(a) 60 m (b) 66 m
(c) 75 m (d) 80 m

Q54. What is the maximum value of 3 sin θ – 4 ?
(a) –4 (b) –1
(c) 0 (d) 1

Q55. If sin θ + cos θ = $\sqrt{2}$, then what is $\sin^6\theta + \cos^6\theta + 6\sin^2\theta\cos^2\theta$ equal to ?
(a) $\frac{1}{4}$ (b) $\frac{3}{4}$
(c) 7 (d) $\frac{7}{4}$

Q56. What is the least value of $9\sin^2\theta + 16\cos^2\theta$?
(a) 0 (b) 9
(c) 16 (d) 25

Q57. If cos 47° + sin 47° = k, then what is the value of $\cos^2 47° - \sin^2 47°$?
(a) $k\sqrt{2-k^2}$ (b) $-k\sqrt{2-k^2}$
(c) $k\sqrt{1-k^2}$ (d) $-k\sqrt{1-k^2}$

Q58. If cosec θ – sin θ = p^3 and sec θ – cos θ = q^3, then what is the value of tan θ ?
(a) $\frac{p}{q}$ (b) $\frac{q}{p}$
(c) pq (d) p^2q^2

Q59. If $0 \le \alpha, \beta \le 90°$ such that cos (α – β) = 1, then what is sin α – sin β + cos α – cos β equal to ?
(a) –1 (b) 0
(c) 1 (d) 2

Q60. Consider the following statements :
1. The value of cos 61° + sin 29° cannot exceed 1.
2. The value of tan 23° – cot 67° is less than 0.
Which of the above statements is/are correct?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

Q61. In a quadrilateral ABCD, ∠B = 90° and $AB^2 + BC^2 + CD^2 - AD^2 = 0$, then what ∠ACD equal to ?
(a) 30° (b) 60°
(c) 90° (d) 120°

Q62. In a ΔABC, AC = 12 cm, AB = 16 cm and AD is the bisector of ∠A. If BD = 4 cm, then what is DC equal to ?
(a) 2 cm (b) 3 cm
(c) 4 cm (d) 5 cm

Q63. ABCD is a cyclic quadrilateral. The bisectors of the angles A, B, C and D. cut the circle at P, Q, R and S respectively. What is ∠PQR + ∠RSP equal to ?
(a) 90° (b) 135°
(c) 180° (d) 270°

Q64. ABC is an equilateral triangle. The side BC is is trisected at D such that BC = 3 BD. What is the ratio of AD^2 to AB^2 ?
(a) 7 : 9 (b) 1 : 3
(c) 5 : 7 (d) 1 : 2

Q65. Consider the following statements :
1. The diagonals of a trapezium divide each other proportionally.
2. Any line drawn parallel to the parallel sides of a trapezium divides the non-parallel sides proportionally.
Which of the above statements is/are correct ?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

Q66. If H, C and V are respectively the height, curved surface area and volume of a cone, then what is $3\pi VH^3 + 9V^2$ equal to ?
(a) C^2H^2 (b) $2\ C^2H^2$
(c) $5\ C^2H^2$ (d) $7\ C^2H^2$

Q67. How many solid lead balls each of diameter 2 mm can be made from a solid lead ball of radius 8 cm ?
(a) 512 (b) 1024
(c) 256000 (d) 512000

Q68. The two sides of a triangle are 40 cm and 41 cm. If the perimeter of the triangle is 90 cm, what is its area ?
(a) 90 cm^2 (b) 135 cm^2
(c) 150 cm^2 (d) 180 cm^2

Q69. The diagonals of a rhombus differ by 2 units and its perimeter exceeds the sum of the diagonals by 6 units. What is the area of the rhombus ?
(a) 48 square units (b) 36 square units
(c) 24 square units (d) 12 square units

Q70. What is the area of a right-angled triangle, if the radius of the circumcircle is 5 cm and altitude drawn to the hypotenuse is 4 cm ?
(a) 20 cm^2 (b) 18 cm^2
(c) 16 cm^2 (d) 10 cm^2

Q71. In a triangle, values of all the angles are integers (in degree measure) Which one of the following ***cannot*** be the proportion of their measures ?
(a) 1 : 2 : 3 (b) 3 : 4 : 5
(c) 5 : 6 : 7 (d) 6 : 7 : 8

Q72. The length of a rectangle is increased by 10% and breadth is decreased by 10%. Then the area of the new rectangle is
(a) neither increased nor decreased
(b) increased by 1%
(c) decreased by 1%
(d) decreased by 10%

Q73. The surface areas of two spheres are in the ratio 1 : 4. What is the ratio of their volumes?
(a) 1:16 (b) 1:12
(c) 1:10 (d) 1 : 8

Q74. The length, breadth and height of a brick are 20 cm, 15 cm and 10 cm respectively. The number of bricks required to construct a wall with dimensions 45 m length, 0.15 m breadth and 3 m height is
(a) 12450 (b) 11250
(c) 6750 (d) None of the above

Q75. If the sum of all interior angles of a regular polygon is twice the sum of all its exterior angles, then the polygon is
(a) Hexagon (b) Octagon
(c) Nonagon (d) Decagon

Q76. A bicycle wheel makes 5000 revolutions in moving 11 km. What is the radius of the wheel? $\left(\text{Assume } \pi = \frac{22}{7}\right)$
(a) 17.5 cm (b) 35 cm
(c) 70 cm (d) 140 cm

Q77. The volumes of two cones are in the ratio 1 : 4 and their diameters are in the ratio 4 : 5. What is the ratio of their heights?
(a) 25 : 64 (b) 16 : 25
(c) 9 : 16 (d) 5 : 9

Q78. In a triangle ABC, if 2 ∠A = 3 ∠B = 6 ∠C, then what is ∠A + ∠C equal to ?
(a) 90° (b) 120°
(c) 135° (d) 150°

Q79. If the perimeter of a circle and a square are equal, then what is the ratio of the area of the circle to that of the square ?
(a) 1 : π (b) 2 : π
(c) 3 : π (d) 4 : π

Q80. The lengths of the sides of a right-angled triangle are consecutive even integers (in cm). What is the product of these integers ?
(a) 60 (b) 120
(c) 360 (d) 480

Q81. A circle is inscribed in a triangle ABC. It touches the sides AB and AC at M and N respectively. If O is the centre of the circle and ∠A= 70°, then what is ∠MON equal to ?
(a) 90° (b) 100°
(c) 110° (d) 120°

Q82. The sum of the squares of sides of a right-angled triangle is 8,450 square units. What is the length of its hypotenuse ?
(a) 50 units (b) 55 units
(c) 60 units (d) 65 units

Q83. A triangle and a parallelogram have equal areas and equal bases. If the altitude of the triangle is k times the altitude of the parallelogram, then what is the value of k ?
(a) 4 (b) 2
(c) 1 (d) $\frac{1}{2}$

Q84. Area of two squares are in the ratio $m^2 : n^4$. What is the ratio of their perimeters ?
(a) m : n (b) n : m
(c) $m : n^2$ (d) $m^2 : n$

Q85. AD is the median of the triangle ABC. If P is any point on AD, then which one of the following is correct ?
(a) Area of triangle PAB is greater than the area of triangle PAC
(b) Area of triangle PAB is equal to area of triangle PAC
(c) Area of triangle PAB is one-fourth of the area of triangle PAC
(d) Area of triangle PAB is half of the area of triangle PAC

Q86. What is the area of a segment of a circle of radius r subtending an angle θ at the centre?
(a) $\frac{1}{2}r^2\theta$
(b) $\frac{1}{2}r^2\,\theta\left(\theta - 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}\right)$
(c) $\frac{1}{2}r^2\left(\theta - \sin\frac{\theta}{2}\cos\frac{\theta}{2}\right)$
(d) $\frac{1}{2}r^2\sin\frac{\theta}{2}\cos\frac{\theta}{2}$

Q87. ABC is a triangle right-angled at C. Let P be any point on AC and Q be any point on BC. Which of the following statements is/are correct?
1. $AQ^2 + BP^2 = AB^2 + PQ^2$
2. AB = 2PQ
Select the correct answer using the code given below :
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2.

Q88. Four circular coins of equal radius are placed with their centres coinciding with four vertices of a square. Each coin touches two other coins. If the uncovered area of the square is 42 cm^2, then what is the radius of each coin ? $\left(\text{Assume } \pi = \frac{22}{7}\right)$
(a) 5 cm (b) 7 cm
(c) 10 cm (d) 14 cm

Q89. The radii of the flat circular faces of a bucket are x and 2x. If the height of the bucket is 3x, what is the capacity of the bucket ? $\left(\text{Assume } \pi = \frac{22}{7}\right)$
(a) $11x^3$ (b) $22x^3$
(c) $44x^3$ (d) $55x^3$

Q90. If p, q, r, s and t represent length, breadth, height, surface area and volume of a cuboid respectively, then what is $\frac{1}{p} + \frac{1}{q} + \frac{1}{r}$ equal to?
(a) $\frac{s}{t}$ (b) $\frac{2t}{s}$
(c) $\frac{s}{2t}$ (d) $\frac{2s}{t}$

Q91. Fifteen candidates appeared in an examination. The marks of the candidates who passed in the examination are 9, 6, 7, 8, 8, 9, 6, 5, 4 and 7. What is the median of marks of all the fifteen candidates?
(a) 6 (b) 6.5
(c) 7 (d) 7.5

Q92. If the yield (in gm) of barley from 7 plots of size one square yard each, were found to be 180, 191, 175, 111, 154, 141 and 176, then what is the median yield ?
(a) 111 gm (b) 154 gm
(c) 175 gm (d) 176 gm

Q93. Which one of the following measures of central tendency will be used to determine the average size of the shoe sold in the shop ?
(a) Arithmetic mean (b) Geometric mean
(c) Median (d) Mode

Q94. When the class intervals have equal width, the height of a rectangle in a histogram represents
(a) Width of the class (b) Lower class limit
(c) Upper class limit (d) Frequency of the class

Q95. The ages of 7 family members are 2, 5, 12, 18, 38, 40 and 60 years respectively. After 5 years a new member aged x years is added. If the mean age of the family now goes up by 1.5 years, then what is the value of x ?
(a) 1 (b) 2
(c) 3 (d) 4

Q96. The mean weight of 100 students in a class is 46 kg. The mean weight of boys is 50 kg and that of girls is 40 kg. The number of boys exceeds the number of girls by
(a) 10 (b) 15
(c) 20 (d) 25

Q97. What is the algebraic sum of the deviations from the mean of a set of values 25, 65, 73, 75, 83, 76, 17, 15, 7, 14 ?
(a) – 1 (b) 0
(c) 1 (d) 2

Q98. The mean of five observations x, x + 2, x + 4, x + 6, x + 8 is m. What is the mean of the first three observations ?
(a) m (b) m – 1
(c) m – 2 (d) m – 3

Q99. What is the median of 2, 4, 6, ..., 100 ?
(a) 48 (b) 49
(c) 50 (d) 51

Q100. The harmonic mean and the geometric mean of two numbers are 10 and 12 respectively. What is their arithmetic mean?
(a) $\frac{25}{3}$ (b) $\sqrt{120}$
(c) 11 (d) 14.4

Solutions

S1. Ans.(c)
Sol. Let N = $x^3 + x^2 + 16$
$\frac{N}{x} = x^2 + x + \frac{16}{x}$
If N is divided by x so is 16 ,the factors of 16 = 1, 2, 4, 8, 16
So, possible value of x = 5

S2. Ans.(b)
Sol. abc = 30
factors of 30 = 1, 2, 3, 5, 6, 10, 15, 30
Total factors = 8
If we fix 1 factor, then selecting one factors = ${}^8C_1$
Only one factor can be repeated i.e. (factor = 1)
Selecting 2 factors out of 7 = ${}^7C_2$
$= \frac{7\times6}{2\times1} = 21$
Out of this (30, 1, 1) (1, 1, 30) will be eliminated so,21 – 2 = 19
So, total = 8 + 19 = 27 Positive integers

S3. Ans.(d)
Sol. $x^2 - 4x - \log_{10} N = 0$, the roots are real
as we know that
$\log_{10} N = K$
$N = 10^K$
If $N = \frac{1}{10^4}$, So, K = - 4
Or $b^2 - 4ac \geq 0$
$(-4)^2 - 4(1)(-\log_{10} N) = 10$
$16 = -4 \log_{10} N$
$\log_{10} N = -4$
$N = (10)^{-4}$

S4. Ans.(b)
Sol.

	MP	SP
8%	100	92
22%	100	78
	10000	7176

Or 1250 : 897
897 units → 2691
1 unit → 3
1250 units → 1250 × 3
= 3750

S5. Ans.(d)
Sol. $I = a^2 + b^2 + c^2$
a, b → consecutive integers
c = ab
Let a = 1, b = 2 and c = 1 × 2 = 2
$I = 1 + 4 + 4 = 9$ Which is $(odd)^2$
Let a = 2, b = 3 and c = 2 × 3 = 6
$I = 4 + 9 + 36 = 49$ Which is $(odd)^2$

S6. Ans.(a)

Sol. N = 23P62971335

Smallest odd composite Number = 9

for 'N' to divisible by 9 its sum of digits must be divisible by 9

N = 41 + P → 5

for P = 5, N is completely divisible.

S7. Ans.(b)

Sol. R = $\frac{1^5+2^5+3^5+4^5+5^5}{4}$

$= \frac{1^5}{4} + \frac{2^5}{4} + \frac{3^5}{4} + \frac{4^4}{4} + \frac{5^5}{4}$

$\downarrow$ 32 $\downarrow$

= R = 1, R = 0, R = 3, R = 0, R = 1

Remainder = 1+ 3 + 1 = 5

$= \frac{5}{4}$ = R = 1

S8. Ans.(c)

Sol. Unit digit = $3^{99} = 3^{4(24)+3} = 3^{4n+3}$ = (1)×7 = 7

S9. Ans.(a)

Sol. LCM (X, Y) = 28 HCF

HCF + LCM = 1740

HCF + 28 HCF = 1740

HCF = 60

So, LCM = 60 × 28 = 1680

LCM × HCF = Product of two numbers

1680 × 60 = 240 × IInd No.

IInd number = 420

S10. Ans.(a)

Sol. $x^n - a^n$ is divisible by x – a, for every "natural number n"

S11. Ans.(a)

Sol. $\frac{17^{200}}{18} = \frac{(18-1)^{2020}}{18} = (-1)^{2020} = +1$

S12. Ans.(c)

Sol. $\frac{1}{1+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{4}} + \ldots\ldots\ldots + \frac{1}{\sqrt{99}+\sqrt{100}}$

If we rationalize one term $\frac{1}{1+\sqrt{2}} \times \frac{1-\sqrt{2}}{1-\sqrt{2}}$

$= \frac{1-\sqrt{2}}{-1} = \sqrt{2} - 1$

following the pattern

$\sqrt{2} - 1 + \sqrt{3} - \sqrt{2} + \sqrt{4} - \sqrt{3} + \ldots\ldots\ldots\ldots + \sqrt{100} - \sqrt{99}$

Every term will be cancelled

expect = - 1 + $\sqrt{100}$ = 9

S13. Ans.(a)

Sol. $x^m = \sqrt[14]{x\sqrt{x\sqrt{x}}}$

$x^m = \sqrt[14]{x\sqrt{x.x^{1/2}}}$

$= \sqrt[14]{x.x^{3/4}}$

$= \sqrt[14]{x^{7/4}}$

$x^m = x^{\frac{7}{4\times14}} = x^{1/8}$

So, m = 1/8

S14. Ans.(c)

Sol. Let S is the sum of product of the ten numbers ±1, ±2, ±3, ±4, ±5, taking two at a time

Therefore S = 1(-1) + 2(-2) + 3(-3) + 4(-4) + 5(-5)

S = - ($1^2 + 2^2 + 3^2 + 4^2 + 5^2$)

$S = -\frac{5\times6\times11}{6}$

S = -55

S15. Ans.(c)

Sol. (110 + 165) = 132 × $\frac{5}{18}$ × t

275 = 132 × $\frac{5}{18}$ t

t = 7.5 sec

S16. Ans.(d)

Sol. R = t = years

SI = $\frac{1}{4}$ P = $\frac{P\times t^2}{100}$

t = 5 years

S17. Ans.(b)

Sol. Total numbers of lines = Page × lines per page

= 60 × n

No. of line when 3 lines are reduced = n - 30

No. of pages when lines are reduced = 70

Total No. of lines = 70(n - 3) = 70n - 210

So, 60n = 70n - 210

n = 21

S18. Ans.(a)

Sol. $\frac{xM \times x\ hrs \times x\ days}{x\ units} = \frac{y\ M \times y\ hrs. \times y\ days}{K}$

K = $x^2\ y^{-3}$

S19. Ans.(b)

Sol. Let d(n) = number of positive divisor of integer n

d(5) = (1, 5) 2 no. of divisor

d(11) = (1, 11) 2 no. of divisor

d(16) = 1, 2, 4, 8, 16 = 5 no. of divisor

d(55) = 1, 5, 11, 55 = 4 no. of divisor

1. d(5) = d(11)
2. d(5) . d(11) = 4 = d(55)
3. d(5) + d(11) = 4 ≠ [(16) = 5]

(b) 1 and 2 only

S20. Ans.(d)

Sol. $A_n = P_n + 1$ where P_n = Product of first n prime numbers

Prime numbers = 2, 3, 5, 7, 11, 13, 17 . . . . . .

$A_1 = 2 + 1 = 3$

$A_2 = (2 \times 3) + 1 = 7$

$A_3 = (2 \times 3 \times 5) + 1 = 31$

$A_4 = (2 \times 3 \times 5 \times 7) = 211$

Hence A_n is always odd have $A_n + 1 \rightarrow$ Even

$A_n + 2 \rightarrow$ odd

S21. Ans.(b)

Sol. Actual weight = 1000 gm

Faulty weight = 800 gm

Error = 1000 - 800 = 200

% profit = $\frac{Error}{Actual\ weight - Error} \times 100$

= $\frac{200}{800} \times 100 = 25\%$

S22. Ans.(a)

Sol. Amount of water in liters will fall = $3 \times 40 \times \frac{2 \times 1000}{60}$

= 4000 m^3

1m^3 = 1000 liters = 4000000 liters

S23. Ans.(d)

Sol. Let CP = 100

then x = $\frac{100-28}{100} = 72$

y = $\frac{100+12}{100} \times 100 = 112$

$\frac{y}{x} = \frac{112}{72} = \frac{14}{9}$ or 14 : 9

S24. Ans.(c)

Sol. $300\left[\frac{1}{x} - \frac{1}{x+15}\right] = 1$

original = $300 \times 15 = x(x + 15)$

Speed $\leftarrow 60 \times 75 = x(x + 15)$

S25. Ans.(a)

Sol. Ratio given = $\frac{1}{3} : \frac{1}{4} : \frac{1}{5}$

LCM of (3, 4, 5) = 60

ratio becomes = $\frac{60}{3} : \frac{60}{4} : \frac{60}{5}$

= $20_{\times 4} : 15_{\times 12} : 12_{\times 12}$

$10_{\times 8}$

= 160 : 180 : 144

Total Profit = 484 unit = 96,800

1 unit = 200

Profit of first, 160 unit = 160 × 200

Person = 32,000

S26. Ans.(d)

Sol. x varies as y, mathematically, x α y

x = ky

1. $x^2 + y^2 = ky^2 + y^2 = y^2 (k^2 + 1)$

$x^2 - y^2 = ky^2 - y^2 = y^2 (k^2 - 1)$

$(x^2 + y^2)$ α $x^2 - y^2$ Given

$\frac{x^2+y^2}{x^2-y^2} = k_1$

$k_1 = \frac{k^2+1}{k^2-1}$

Hence proved that

$x^2 + y^2 = \frac{k^2+1}{k^2-1} x^2 - y^2$

2. $\frac{x}{y^2}$ varies inversely as y

mathematically $\left(\frac{x}{y^2} \alpha \frac{1}{y}\right)$

$\frac{x}{y^2} = \frac{ky}{y^2} = \frac{k}{y}$ Hence proved

3. $\sqrt[n]{x^2 y}$ varies as $\sqrt[2n]{x^4\ y^4}$

$\sqrt[n]{x^2 y} \times \sqrt[2n]{x^4\ y^4}$

Put x = ky

$\sqrt[2n]{x^4\ y^2} = \sqrt[2n]{x^4\ y^4} = \sqrt[n]{x^2\ y}$

So, $\sqrt[n]{x^2\ y}$ varies as $\sqrt[2n]{k^2\ y^4}$

S27. Ans.(c)

Sol. Ena's age = 13

Ena's mother age = 13 + 24 = 37

Ena's father age = 37 + 3 = 40

Ena's father's age when they got married = 40 – (13 + 4 = 17) = 23

S28. Ans.(b)

Sol. Mahesh's age = 60 yr

Ram's age = 60 - 5 = 55 years

Raju's age = 55 – 4 = 51 yr

Babu's age = 51 – 6 = 45 yr

Diff. (Mahesh - Babu) = 60 - 45 = 15 yr.

S29. Ans.(a)

Sol. Let number of items in a booklet N = 100

x% = 10%

$$100 \xrightarrow[I^{st}\ yrs]{+\ 10\ \%} 110 \xrightarrow[II^{nd}\ yr]{-\ 10\%} 99$$

its always less than N

S30. Ans.(c)

Sol. ab + xy – xb = $\Longrightarrow$ x(y - b) = - ab

bc + yz - cy = 0 $\Longrightarrow$ c(b - y) = - yz

$\frac{x}{a} + \frac{c}{z} = \frac{-ab}{(y-b)\times a} + \frac{-yz}{(b-y)\times z}$

= $\frac{-b}{b-y} + \frac{-y}{b-y}$

= $\frac{b-y}{b-y} = 1$

S31. Ans.(d)

Sol. f(x) = $x^6 - 3x^4 + 3x^2 - 1$

= $x^6 - x^4 - 2x^4 + 2x^2 + x^2 - 1$

= $x^4\ (x^2 - 1) - 2x^2(x^2 - 1) + 1\ (x^2 - 1)$

= $(x^2 - 1)\ (x^4 - 2x^2 + 1)$

= $(x + 1)\ (x - 1)\ (x^2 - 1)^2$

= $(x + 1)\ (x - 1)\ (x - 1)^2\ (x + 1)^2 = (x + 1)^3\ (x - 1)^3$

$g(x) = x^3 + 3x^2 + 3x + 1$

$= (x + 1)^3$

Now, HCF (f(x), g(x) = $(x + 1)^3$

S32. Ans.(d)

Sol. HCF × LCM = Product of two numbers

$(3x + 1) \times (30x^3 + 7x^2 - 10x - 3) = (6x^2 + 5x + 1) \times II^{nd}$

$30x^3 + 7x^2 - 10x - 3 = 2x + 1 \times II^{nd}$

$15x^2 - 4x - 3 = II^{nd}$

S33. Ans.(c)

Sol. (p + 2) (2q - 1)= 2pq – 10

2pq – p + 4q – 2 = 2pq – 10

4q – p + 8 = 0 _____(1)

(p – 2) (2q - 1) = 2pq – 10

2pq – p - 4q + 2 = 2pq – 10

4q + p - 12 = 0_____(2)

Now, add equation (1) and (2)

4q – p + 8 + 4q + p – 12 = 0

q = ½

put q = ½ in equation (1),

4(1/2) – p + 8 = 0

P = 10

Now pq = 10(1/2) = 5

S34. Ans.(d)

Sol. $\frac{a^2+ac}{a^2c-c^3} - \frac{a^2-c^2}{a^2c+2ac^2+c^3} - \frac{2c}{a^2-c^2} + \frac{3}{a+c}$

$\frac{a\ (a+c)}{c\ (a^2-c^2)} - \frac{a^2-c^2}{c(a^2+2ac+c^2)} - \frac{2c}{a^2-c^2} + \frac{3}{a+c}$

$\frac{a}{c(a-c)} - \frac{a-c}{c(a+c)} - \frac{2c}{a^2-c^2} + \frac{3}{a+c}$

$\frac{a(a+c) - (a-c)^2 - 2c^2 + 3c(a-c)}{c\ (a^2-c^2)}$

$\frac{a^2+ac-a^2-c^2+2ac-2c^2+3ac-3c^2}{c(a^2-c^2)}$

$\frac{6ac-6c^2}{c(a^2-c^2)} = \frac{6c\ (a-c)}{c(a-c)(a+c)} = \frac{6}{a+c}$

S35. Ans.(d)

Sol. $4x^4 + 8x^3 - 4x + 1 + 4x^2 - 4x^2$

= $(2x^2)^2 + (2x)^2 + (1)^2 + 2 \times 2x^2 \times 2x - 2 \times 2x \times 1 - 2 \times 2x^2 \times 1$

= $(2x^2 + 2x - 1)^2$

So, $\sqrt{4x^4 + 8x^3 - 4x + 1} = \sqrt{(2x^2 + 2x - 1)^2}$

= $2x^2 + 2x - 1$

S36. Ans.(b)

Sol. Let the digits be x and y $\Longrightarrow$ x + y = 13

and N = 10x + y

N_1 = 10y + x

9(x - y) = 27

x – y = 3

x = 8, y = 5

Product of digits = xy = 40

S37. Ans.(b)

Sol. $\frac{x}{b+c} = \frac{y}{c+a} = \frac{z}{b-a} = k$

x = k(b + c)

y = k(c + a)

z = k(b - a)

x – y – z = 0

k[b + c – c – a – b + a]

k[0] = 0

S38. Ans.(b)

Sol. Distance covered by x in 2 hours = 4 × 2

= 8 km

Time required for y to cover this distance = $\frac{8}{5-4}$

Let = starts t hour after y

8 × 5 = (8 - t) × 6

t = 4/3 hours

S39. Ans.(b)

Sol. Let N = 1 – x - $x^n + x^n\ .x$

= 1.(1 – x) – $x^n\ (1 - x)$

= $(1 - x^n)(1 - x)$

If n = 1, then N = (1 - x) (1 - x) = $(1 - x)^2$

n = 2 then N = (1 - x) (1 + x) (1 - x) = $(1 - x)^2$ (1 + x)

n = 3 then N = $(1 – x^3)$ (1 - x) = $(1 - x)^2$ .....

then clearly it is divisible by $(1 - x)^2$

S40. Ans.(c)

Sol. CP = x

Profit = x%

$x \times \frac{100+x}{100} = 75$

$x(100 + x) = 7500$

at x = 50, the above eq^n will satisfy 50%

S41. Ans.(b)

Sol. Let average speed = S

total time = t

Hence D = st

If speed = s + x

time = t – y

D = (s + x) (t - y)

st = st – sy + x(t - y)

sy = x (t - y)

$s = x(t - y)y^{-1}$

So distance = st

$= x(t - y)ty^{-1}$

S42. Ans.(a)

Sol. Time $\alpha \sqrt{Distance}$

$Time^2 = k \times Distance$

$\frac{(4^2) = k \times 78.40}{(t)^2 \quad k \times 122.50}$

$t^2 = \frac{122.50}{78.40} \times 16$

$t^2 = \frac{1225}{49}$

$t = \frac{35}{7} = 5$ sec

S43. Ans.(b)

Sol. $6^{3-4x}.4^{x+5} = 8$

$3^{3-4x}.2^{3-4x}.2^{2x+10} = 2^3$

$3^{3-4x}.2^{13-2x} = 2^3$

$3^{3-4x} = 2^{2x-10}$

Taking log both sides

$\log 3^{3-4x} = \log 2^{2x-10}$

$(3 – 4x) \log 3 = (2x – 10) \log 2$

$(3 – 4x) (0.477) = 2(x - 5) (0.301)$

$1.431 – 1.908x = 0.602x – 3.01$

$2.51x = 4.441$

$x = \frac{4.441}{2.51} = 1.7693$

$1 < x < 2$

S44. Ans.(d)

S45. Ans.(d)

Sol. If radius = $\frac{a}{b}$ where b ≠ 0

surface area = $4\pi \frac{a^2}{b^2}$

Presence of π irrational will always make them irrational.

S46. Ans.(b)

Sol. $\operatorname{cosec} \theta - \sin \theta = m$

$\frac{1}{\sin\theta} - \sin\theta = m$

So, $m = \frac{cos^2\theta}{\sin\theta}$ _____(1)

$\sec \theta - \cos \theta = n$

$\frac{1}{\cos\theta} - \cos \theta = n$

$\frac{sin^2\theta}{\cos\theta} = n$ _____(2)

$m^{\frac{4}{3}} n^{\frac{2}{3}} + m^{\frac{2}{3}} n^{\frac{4}{3}}$

$(m^2 n)^{\frac{2}{3}} + (mn^2)^{\frac{2}{3}}$

$\left(\frac{\cos^4\theta}{sin^2\theta} \times \frac{sin^2\theta}{\cos\theta}\right)^{2/3} + \left(\frac{\cos^2\theta}{sin\theta} \times \frac{sin^4\theta}{\cos^2\theta}\right)^{2/3}$

$cos^2\theta + sin^2\theta = 1$

S47. Ans.(b)

Sol. $\cos \theta + \sec \theta = k$

Squaring both side

$\cos^2 \theta + \sec^2 \theta + 2 = k^2$

$\cos^2 \theta + \sec^2 \theta = k^2 - 2$

value = $\sin^2 \theta – \tan^2 \theta$

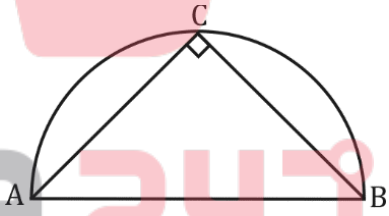
$= 1 – \cos^2 \theta – (\sec^2 \theta - 1)$

$= 2 – \cos^2 \theta - \sec^2 \theta$

$= 2 – k^2 + 2 = 4 – k^2$

S48. Ans.(d)

Sol.



Angle made by a diameter on any part of circle is 90°

= cos (A + B) + sin (A + B)

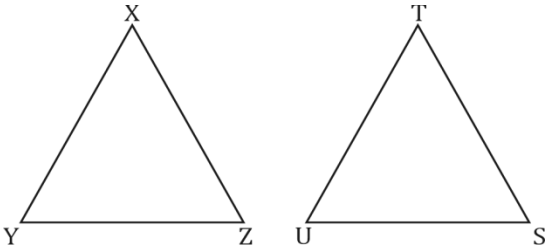
= cos (90 – c) + sin (90 - c)

= sin c + cos c (∠C = 90)

= 1

S49. Ans.(a)

Sol.



$\frac{ar(\Delta XYZ)}{ar(\Delta TUS)} = \left(\frac{YZ}{US}\right)^2$

$\Rightarrow \frac{25}{16} = \left(\frac{10}{US}\right)^2 \Rightarrow US = 8cm.$

S50. Ans.(c)

Sol. Exterior angle of pentagon = $\frac{360}{5}$ = 72

I + E = 180

I = 108°

180° → π radian

108° → $\frac{108}{180}$ π rad = $\frac{3\pi}{5}$

S51. Ans.(b)

Sol. X – Y = 15°

X + Y = $\frac{5\pi}{12}$ = 75°

X = 45, Y = 30

45 = ? × 30

? = $\frac{3}{2}$

S52. Ans.(b)

Sol. St 1:-

$2\sin^2\theta - \cos\theta + 4 = 0$

$2(1 - \cos^2\theta) - \cos\theta + 4 = 0$

$2 - 2\cos^2\theta - \cos\theta + 4 = 0$

$2\cos^2\theta + \cos\theta - 6 = 0$

$2\cos^2\theta + 4\cos\theta - 3\cos\theta - 6 = 0$

$\cos\theta = \frac{3}{2}$ or -2

Not possible $-2 \le \cos\theta \le 1$

St 2:-

$\tan\theta + \cot\theta = 2$

$\frac{\tan+\cot\theta}{2} \ge \sqrt{\tan\theta}\ .\cos\theta$ (AM ≥ GM)

$\tan\theta + \cot\theta \ge 2$

St 2 is correct.

S53. Ans.(a)

Sol. We know that

length of an arc = radius × angle

$44 = r \times 42 \times \frac{\pi}{180}$

$44 = r \times 42 \times \frac{22}{7\times180}$

r = 60 m

S54. Ans.(b)

Sol. max. value = $3\sin\theta - 4$

$-1 \le \sin\theta \le 1$

max value = 3(1) – 4 = -1

S55. Ans.(d)

Sol. $\sin\theta + \cos\theta = \sqrt{2}$

at ($\theta = 45°$)

$\sin^6\theta + \cos^6\theta + 6\sin^2\theta\cos^2\theta$

$\sin^6 45° + \cos^6 45° + 6\sin^2 45°\cos^2 45°$

$\frac{1}{8} + \frac{1}{8} + 6 \times \frac{1}{2} \times \frac{1}{2} = \frac{7}{4}$

S56. Ans.(b)

Sol. $9\sin^2\theta + 16\cos^2\theta$

$=9\sin^2\theta + 16 - 16\cos^2\theta$

$= -7\sin^2\theta + 16$ ($\sin\theta = -1$)

=9

S57. Ans.(a)

Sol. $\cos 47 + \sin 47 = k$

Squaring both sides

$\cos^2 47 + \sin^2 47 + 2\sin 47\cos 47 = k^2$

$\sin 94 = k^2 - 1$

$(\cos 47 - \sin 47)^2 = \cos^2 47 + \sin^2 47 - 2\sin 47\cos 47$

$= 1 - \sin 94$

$= 1 - k^2 + 1$

$\cos 47 - \sin 47 = \sqrt{2 - k^2}$

$\cos^2 47 - \sin^2 47 = (\cos 47 + \sin 47)(\cos 47 - \sin 47)$

$= k\sqrt{2 - k^2}$

S58. Ans.(b)

Sol. $\text{cosec}\,\theta - \sin\theta = p^3 \Rightarrow \frac{1-\sin^2\theta}{\sin\theta} = p^3$

$\sec\theta - \cos\theta = q^3 \Rightarrow \frac{1-\cos^2\theta}{\cos\theta} = q^3$

$\frac{\cos^2\theta}{\sin\theta} = p^3$ $\quad 1\,\frac{\sin^2\theta}{\cos\theta} = q^3$

$\frac{\cos^2\theta}{\sin\theta} \times \frac{\cos\theta}{\sin^2\theta} = \frac{p^3}{q^3}$

$\left(\frac{\cos\theta}{\sin\theta}\right)^3 = \left(\frac{p}{q}\right)^3$

$\tan\theta = \frac{q}{p}$

S59. Ans.(b)

Sol. $\cos(\alpha - \beta) = 0$ $\quad 1(\cos 0)$

$\alpha - \beta = 0$

$\alpha = \beta$

$\sin\alpha - \sin\beta + \cos\alpha - \cos\beta = 0$

S60. Ans.(a)

Sol. St 1

$\cos (90 - 29) + \sin 29 = 2 \sin 29$

Since, $\sin 30 = 0.5$

$2 \sin 29 < 1$

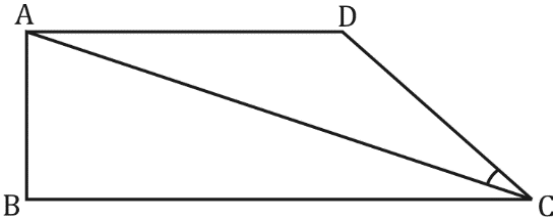
St 2

$\tan 23 - \cot 67 =$

$\tan 23 - \cot (90 - 67) = 0$

S61. Ans.(c)

Sol.



$AB^2 + BC^2 + CD^2 - AD^2 = 0$

$(AC^2) + CD^2 - AD^2 = 0$..... One more Pythagoras

$AC^2 + CD^2 = AD^2$..... (1)

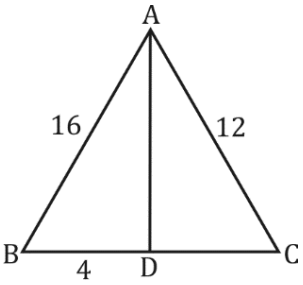
Here, equation (1) is the equation from of Pythagoras theorem applied in ΔACD.

For equation (1) true, the side AD must be the hypotenuse of the ΔACD

means $\angle ACD = 90°$

S62. Ans.(b)

Sol.

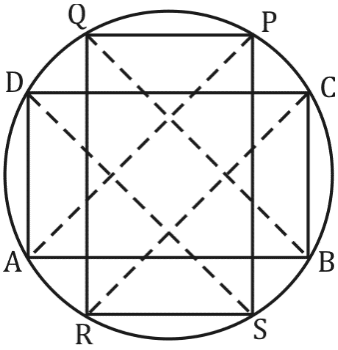


Acc. to internal bisector theorem

$\frac{AB}{BD} = \frac{AC}{CD} \Rightarrow \frac{16}{4} = \frac{12}{CD}$ CD = 3

S63. Ans.(c)

Sol.



Bisector of A, B, C and D cut at P, Q, R and S respectively.

Here, by joining the point P and B we get PR as diameter of the circle.

Now, PQR is a triangle made in the semi-circle PQR by taking PR as the base of the triangle & PSR is a triangle made in the semi-circle PSR by taking PR as the base of the triangle.

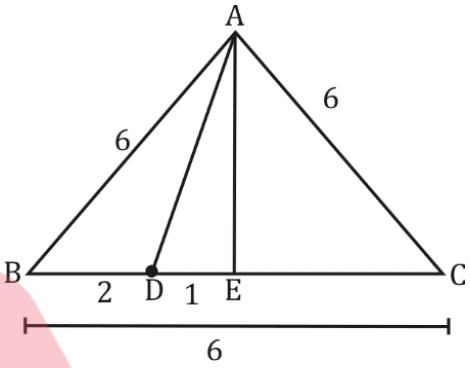
Then, $PQR = PSR = 90^0$

$PQR + PSR = 90 + 90^0$

$\angle PQR + \angle RSP = 180°$

S64. Ans.(a)

Sol.



In ΔAEB

$BE = BC/2$

$DE = \frac{BC}{2} - \frac{BC}{3} = \frac{BC}{6}$

$AD^2 = DE^2 + AE^2$

$= 1 + \left(\frac{\sqrt{3}}{2} \times 6\right)^3 = 28$

$\frac{AD^2}{AB^2} = \frac{28}{6 \times 6} = \frac{7}{9}$ Or 7 : 9

S65. Ans.(c)

Sol. Statement 1

In trapezium WXYZ in which ZX and WY are two diagonals and PQ||WX||ZY

In ΔWZY, PQ||ZY so

WP/PZ = WQ/ QY .....(1)

In ΔWZX, PQ || WX,

ZP/ PW = ZQ / QX or PW/ZP = QX/ZQ .....(2)

From equation (1) and (2)

WQ/QY = QX/ ZQ

So proved that diagonals divide each other proportionally.

Statement 2

In trapezium WXYZ in which ZX is a diagonal and PR||WX||ZY and PQ||ZY||WX

In ΔZWX PR || WX

ZP/ PW = RZ/XR.....(1)

In ΔXYZ, RQ/ZY

XQ/QY = XR/RZ or QY/XQ =RZ / XR .....(2)

From equation (1) and (2)

ZP/PW = YQ/QX

Proved that any line drawn parallel to the parallel lines divides non parallel line proportionally.

S66. Ans.(a)

Sol. $=3\pi \times \frac{1}{3} r^2 H \times H^3 + 9 \times \frac{1}{9} \pi^2 r^4 H^2$

$=\pi^2 r^2 H^4 + \pi^2 r^4 H^2$

$=\pi^2 r^2 H^2 [H^2 + r^2]$

$=\pi^2 r^2 H^2 L^2$ $[L^2 = H^2 + r^2]$

$=(\pi r L)^2 H^2$

$=C^2 H^2$

S67. Ans.(d)

Sol. Given,

Radius of lead ball = 8cm

Let number of lead ball of each 2mm diameter = N

1cm = 10 m

$\frac{4}{3}\pi \times 8 \times 8 \times 8\ cm^3 = n \times \frac{4}{3}\pi \times 1 \times 1 \times 1\ mm^3$

$512 \times 10 \times 10 \times 10 = n$

N = 5,12,000

S68. Ans.(d)

Sol. Given,

Perimeter of triangle = 90cm

Sides of Triangle a and b are 40cm and 41cm respectively.

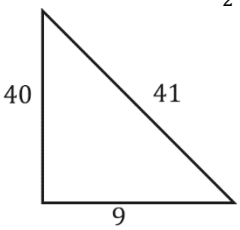
Perimeter of triangle = a + b + c

90cm = 40cm +41cm + c

C = 9cm

The side of the triangle are in triplets, height, base and hypotenuse are 40cm, 9cm and 41cm respectively.

Area of triangle = $\frac{1}{2}$ × height × base



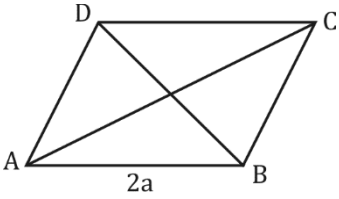
Ar = $\frac{1}{2} \times 40 \times 9 = 180$

S69. Ans.(c)

Sol. Given,

Diagonals of a rhombus differ by = 2 units

Perimeter exceeds the sum of the diagonals by = 6 units



AC – BD = 2 units

4a = (AC + BD) + 6

Let side = 2a

Area of rhombus = $\frac{d_1 \times d_2}{2}$

Side = $\sqrt{\left(\frac{d_1}{2}\right)^2 + \left(\frac{d_2}{2}\right)^2}$

If $d_1 = 2x$ then $d_2 = 2(x + 1)$

side = $\sqrt{x^2 + (x + 1)^2}$

Perimeter of rhombus = sum of diagonal + 6

$4\sqrt{x^2 + (x + 1)^2} = 4x + 8$

Squaring both sides

$16[x^2 + (x + 1)^2] = 16(x + 2)^2$

$x^2 + x^2 + 1 + 2x = x^2 + 4 + 4x$

$x^2 - 2x - 3 = 0$

$(x - 3)(x + 1) = 0$

x = 3

So, length of $d_1 = 6$, $d_2 = 8$

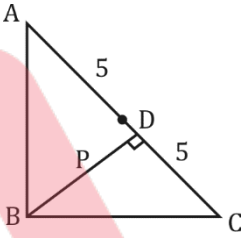
Ar. = $\frac{6 \times 8}{2} = 24\ cm^2$

S70. Ans.(a)

Sol. Given,

Radius of the circumcircle = 5 cm

Height of the altitude = 4cm



p = $\frac{AB \times BC}{AC}$

4 = $\frac{AB \times BC}{10}$

AB × AC = 40

Area of Δ ABC = $\frac{1}{2}$ × AB × AC = 20 cm^2

S71. Ans.(d)

Sol. For every angle integer value the sum of the ratio of three angles must completely divide 180. All except option d.

S72. Ans.(c)

Sol. Area of the rectangle = length × breadth

Let the length and breadth of the rectangle are 100L and 100B respectively.

100 L × 100 B = 10000

↓ +10% ↓ -10%

Area = 110 L × 90 = 9900

It will decreased = $\frac{100}{10000} \times 100 = 1\%$

S73. Ans.(d)

Sol. Surface area of the spheres $=4\pi r^2$

$\frac{4\pi r_1^2}{4\pi r_2^2} = \frac{1}{4}$

$\frac{r_1}{r_2} = \frac{1}{2}$ Or 1 : 2

$\frac{r_1^3}{r_2^3} = \frac{1}{8}$ Or 1 : 8

S74. Ans.(c)

Sol. Given,

Length breadth and height of brick are 20cm, 15cm and 10cm respectively.

Let the number of bricks required = n

No. of bricks = $\frac{dimesions\ of\ wall}{dimensions\ of\ brick}$

$= \frac{45\ m \times 0.15\ m \times 3\ m \times 100 \times\ 100 \times\ 100}{20\ cm \times 15\ cm \times 10\ cm}$

= 6750

S75. Ans.(a)

Sol. Exterior angle = $\frac{360}{n}$

n × Exterior angle = 360 [Sum of Exterior angle]

Sum of interior angle = (2n - 4) × 90

= (n - 2) × 180

ATQ,

(n - 2) × 180 = 2 × 360

n = 6 (Hexagon)

S76. Ans.(b)

Sol. 1 Revolution = 2 π r

n Revolution = n × 2 π r

n = 5000 distance covered = 11 km

$5000 \times 2 \times \frac{22}{7} \times r = 11 \times 1000$

$r = \frac{7}{20}$ m

= 35 cm (1m = 100 cm)

S77. Ans.(a)

Sol. Volume of the cone = $\frac{1}{3}\ \pi\ r^2\ h$

$\frac{V_1\ of\ cone\ 1}{V_2\ of\ cone\ 2} = \frac{1}{4}$

$\frac{\frac{1}{3}\pi\ r_1^2\ h_1}{\frac{1}{3}\pi\ r_2^2\ h_2} = \frac{1}{4}$

$\frac{16\ h_1}{25\ h_2} = \frac{1}{4}$

$\frac{h_1}{h_2} = \frac{25}{64}$

S78. Ans.(b)

Sol. To get the ratio of angle divide 2 ∠A = 3 ∠B = 6 ∠C by, the LCM of 2, 3 and 6 respectively.

The ratio of angle are 3 : 2 : 1 of angle A, B and C respectively.

2∠A = 3∠B = 6∠C = k

$\angle A = \frac{k}{2}, \angle B = \frac{k}{3}, \angle C = \frac{k}{6}$

$\frac{k}{2} + \frac{k}{3} + \frac{k}{6} = 180$

$\frac{3k+2k+k}{6} = 180$

k = 180

∠A = 90°, ∠B = 60°, ∠C = 30°

∠A + ∠C = 90 + 30 = 120°

S79. Ans.(d)

Sol. Perimeter of circle = Perimeter of square

2π r = 4 × a

r = 2a/π

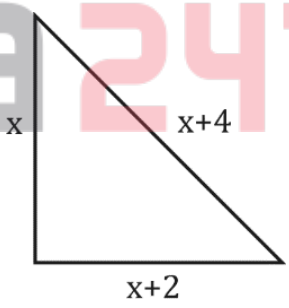
$\frac{Ar.of\ circle}{Ar.of\ square} = \frac{\pi r^2}{a^2}$

$= \frac{\pi \times 4a^2}{\pi^2 \times a^2}$

$= \frac{4}{\pi}$ Or 4 : π

S80. Ans.(d)

Sol. Let the perpendicular, base and hypotenuse are x, x + 4, and x + 2 respectively.



Let three even numbers be x, x + 2 & x + 4

$(x + 4)^2 = (x^2) + (x + 2)^2$

$x^2 + 16 + 8x = x^2 + x^2 + 4 + 4x$

$x^2 - 4x - 12 = 0$

$x^2 - 6x + 2x - 12 = 0$

x(x - 6) + 2 (x - 6) = 0

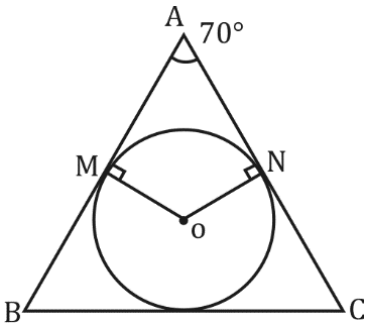
x = 6

Now, sides are (6, 8, 10)

their product of integers = 6 × 8 × 10 = 408

S81. Ans.(c)

Sol.



In quad ONAM

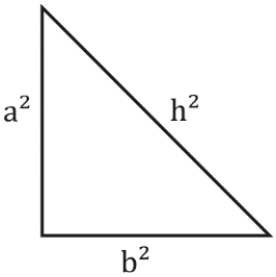
$\angle O + \underbrace{\angle N + \angle A} + \angle M = 360°$

$\angle O + \quad 180° \quad + 70 = 360°$

$\angle O = 110°$

S82. Ans.(d)

Sol. Given, sum of the square of a right-angle triangle is 8450 square units.



$a^2 + b^2 = h^2$ (Pythagoras)

$a^2 + b^2 + h^2 = 8450$

$2h^2 = 8450$

$h^2 = 4225$

$h = \sqrt{4225} = 65$

S83. Ans.(b)

Sol. Let the length of the base of a triangle and a parallelogram be b.

ATQ

Base of triangle = base of parallelogram = b

If 'a' is the altitude of ||gm

then 'ka' is altitude of Δ

b = base of parallelogram and Δ

Area of parallelogram = Area of triangle,

$ba = \frac{1}{2} \times ka \times b$

$k = 2$

S84. Ans.(c)

Sol. Let side of square m = a

side of square n = b

ATQ,

$\frac{a^2}{b^2} = \frac{m^2}{n^2}$

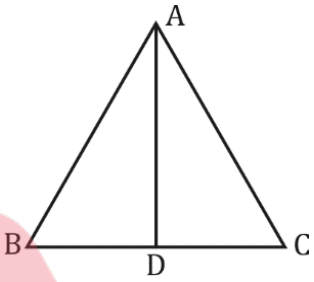
$\frac{a}{b} = \frac{m}{n}$

ratio of perimeters $\frac{4a}{4b} = \frac{4m}{4n^2} = \frac{m}{n}$ or m : n^2

0

S85. Ans.(b)

Sol.



In ΔABC, AD is the median of ΔABC

Area of ΔABC = Area of ΔADC…….(1)

In ΔPBC PD is also the median of ΔPBC

Area of ΔPBD = area of ΔPCD ……..(2)

Subtracting equation (2) from (1)

Area of ΔABD – Area of ΔPBD = Area of ΔADC – Area of ΔPCD

Area of ΔPAB = Area of ΔPAC

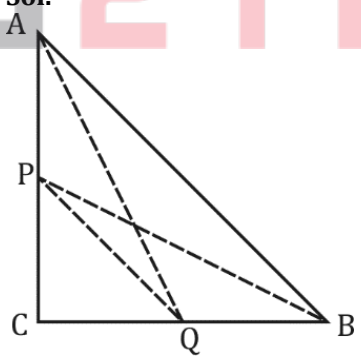
S86. Ans.(a)

Sol. The area of a segment of a circle of radius r subtending an angle θ at the centre = $\frac{1}{2}r^2\theta$

The

S87. Ans.(c)

Sol.



$AQ^2 = AC^2 + QC^2$

$BP^2 = PC^2 + BC^2$

$AQ^2 + BP^2 = AC^2 + BC^2 + PC^2 + QC^2$

$AQ^2 + BP^2 = AB^2 + PQ^2$ ……. St 1 ✓

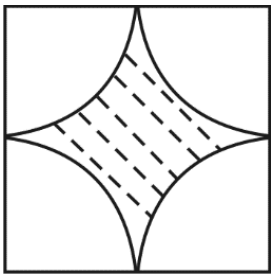
If P and Q be any point, then they can cut them into two equal length

then Δ PCQ ~ Δ ACB

and AB = 2PQ ……. St 2 ✓

S88. Ans.(b)

Sol.



Let the side of the square be 'a' cm and r be the radius of each circle.

Side of square = r + r =2r

a = 2r

ATO

Uncovered area of square = area of square – [4(1/4) area of a circle]

This shaded region Area = 42 cm^2

$\frac{3}{14}a^2 = 42$

$3a^2 = 14 \times 14$

a = 14

Radius of each coin = r = a/2 = 7

S89. Ans.(b)

Sol. Vol. of bucket = Vol. of frustum

$= \frac{1}{3}\pi\ (r^2 + R^2 + Rr) \times h$

$= \frac{1}{3} \times \frac{22}{7} \times 7x^2 \times 3x = 22x^3$

S90. Ans.(a)

Sol. p = length, q = breadth, r = height

s = Surface Ar. of cuboid

= 2(pq + qr + rp)

t = Volume of cuboid

= pqr

value of $= \frac{1}{p} + \frac{1}{q} + \frac{1}{r}$

$= \frac{qr+rp+pq}{pqr} = \frac{s}{t}$

S91. Ans.(a)

Sol. $(a < a_2 < a_3 < a_4 < a_5)$

Let the marks of those failed be $a_1,\ a_2,\ a_3,\ a_4, a_5$

Marks of candidates who passed are given as a, 6, 7, 8, 8, 8, 9, 6, 5, 4 and 7

Now arrange them in ascending order

$a_1,\ a_2,\ a_3,\ a_4, a_5$, 4, 5, 6, 6, 7, 7, 8, 8, 9, 9

for odd number, median = $\left(\frac{n+1}{2}\right)^{th}$ term

$= \left(\frac{15+1}{2}\right)$ = 8th term

median of marks = 8th term = 6

S92. Ans.(c)

Sol. Arrange them in ascending order

111g, 141g, 154g. 175g, 176gm, 180g and 191gm

median = mid – number = $\left(\frac{7+1}{2}\right)^{th}$ = 4th term

median of yield = 175 gm

S93. Ans.(d)

Sol. Mode

S94. Ans.(d)

Sol. Frequency of the class

S95. Ans.(b)

Sol. Mean = $\frac{Sum\ \ of\ \ observations}{Number\ \ of\ \ observations}$

$= \frac{2 + 5 + 12 + 18 + 38 + 40 + 60}{7}$

$= \frac{175}{7} = 25$

After 5 years, their means = 30

but After 5 years, their mean increased by 1.5

Hence, mean = 26.5 for 8 members

Total = 212

So, age is = Actual Increased – what should have increased

= 212 - 210

= 2

S96. Ans.(c)

Sol. Total weight of 100 students = 100 × 46 = 4600

Let no. of boys be x

then total age of boys = x × 50

total age of girls = (100 – x) × 40

50x + 4000 - 40x = 4600 $\Longrightarrow$ x = 60

No. of boys exceed the no. of girls by 60 – 40 = 20

S97. Ans.(b)

Sol. Mean = $\frac{25+65+73+75+83+76+17+15+7+14}{10}$

$= \frac{450}{10}$

= 45

25

Deviation from 45 of 25 = 45 – 25 = + 20

= 45 – 65 = - 20

= 45 -73 = - 28

= 45 – 75 = - 30

= 45 – 83 = - 38

= 45 – 76 = - 31

= 45 – 17 = 28

= 45 – 15 = 30

= 45 – 7= 38

= 45 – 14 = 31

Sum of deviations = 0

S98. Ans.(c)

Sol. Mean = $\frac{x+x+2+x+4+x+6+x+8}{5}$ = m

$\frac{5x+20}{5}$ = x + 4 = m

Mean of first three of observation

mean = $\frac{x+x+2+x+4}{3} = \frac{3(x+2)}{3}$ = x + 2

= m – 4 + 2

= m - 2

S99. Ans.(c)

Sol. Median, 2, 4, 6, ... 100

a_n = a + (n - 1) d

100 = 2 + 2n – 2

n = 50

Median of even observation = n/2 = 25th term

a_{25} = a + 24 d

= 2 + 24 × 2 = 50

S100. Ans.(d)

Sol. (Geometric mean)^2 = Harmonic mean × Arithmetic mean

$(12)^2$ = 10 × Arithmetic mean

Arithmetic mean = 14.4