U.P. EDUCATION SERVICE SELECTION COMMISSION, PRAYAGRAJ

Syllabus

STATISTICS

(Assistant Professor-Higher Education)

(Subject Code-77)

Unit-1 : Probability Theory :

Random experiment: Trial, sample point, sample space, definitions of equally likely, mutually exclusive and exhaustive events, events class, definition of probability, classical and relative frequency approach to probability, axiomatic approach to probability and its properties, merits and demerits of these approaches, total and compound probability theorems, conditional probability, independence of events, Bayes' theorem and its applications.

Unit-2 : Distribution Theory :

Random Variable: Concept of discrete random variable, probability mass function and distribution function, joint probability mass function of several discrete random variables, marginal and conditional probability mass functions. Continuous random variable: Probability density function, distribution function, joint density function of two continuous variables, marginal and conditional probability density functions, independence of random variables. Mathematical expectation, moments, moment generating function of a random variable, their properties and uses, probability generating function, Chebyshev's inequality and its applications, basic ideas of convergence in probability and convergence in distribution, Markov's inequality,

Weak Law of large numbers (WLLN) and Central limit theorem (CLT) for a sequence of independently and identically distributed random variables and their applications. functions of random variables. Bernoulli distribution, binomial distribution, Poisson distribution, geometric distribution, negative binomial distribution, hyper geometric distribution, multinomial distribution, uniform distribution, normal distribution and its relationship with the binomial and Poisson distribution, Exponential, Gamma, beta, Cauchy distribution, Laplace, pareto, weibull, log hormal distributions. bivariate normal distribution and its marginal and conditional distributions.

Sampling distribution of a statistic, Derivation of χ^2, t, F and Z distributions, Beta Gamma and Laplace densities.

Unit-3 : Statistical Methods :

Concept of Statistical population, Attributes and variables (discrete and Continuous), Different types of scales-nominal, ordinal, ratio and interval, Primary data, designing a questionnaire and Schedule, Collection of primary data, checking their consistency, Secondary data, Scrutiny of data for internal consistency and detection of errors of recording, Ideas of cross validation, Presentation of data, classification, tabulation, diagrammatic and graphical representation of grouped data, Frequency distribution, Cumulative frequency distributions and their graphical representation, histogram, frequency polygon and ogives, stem and leaf Plot, Box plot. Concept of central tendency and its measures, partition values, dispersion and relative dispersion, moments, Sheppard's correction for moments (without derivation), skewness, kurtosis and their measures. Measures based on quartiles

Bivariate data: Scatter diagram, product moment correlation coefficient and its properties, coefficient of determination, correlation ratio, intra class correlation, and

concept of error in regression, principle of least square, fitting of linear regression equations and related results, rank correlation.

Partial and multiple correlation in three variables, their measures and related results.

Theory of attributes: Independence and Association of attributes, Karl Pearson's and Tchuprow's Coefficient of association for two way classified data, contingency table in the ordered categories.

Unit-4 : Statistical Quality Control and Time Series :

Central Chart for variables and attributes, modified control charts, group control chart, CUSUM charts, V-mask. Control chart : p- chart, np chart, c chart, chart for variables : (X, R), ($\bar{X}$, σ) charts. Sampling inspection by attributes - single and double samplings plans, producer's and consumer's risk, OC, ASN, ATI fucntions, AOQL, LTPD of sampling plans- sampling inspection by variables- sample cases.

Time Series Analysis: time series and its different components, illustration, additive and multiplicative models, determination of trend, seasonal and cyclical fluctuations, growth curves, analysis of seasonal fluctuations, construction of seasonal indices, idea of correlogam and peridogram.

Unit-5 : Demography and Index Number :

Demographic methods: source of demographic data- census, register, adhoc survey, hospital records, demographic profile of Indian censuses.

Measurement of mortality crude, death rates, age specific death rates standardized death rates infant mortality rates (IMR) , death rates by cause.

Measurement of Fertility- Cruds Birth rate, General fertility rate, age-specific fertility rate, total fertility rate, Gross reproduction, rate, net reproduction rate, age pyramid of sex composition, others measures of fertility. Graduation of mortality rate by Gompertz and Makhan formula, logistic curve fitting and its use in population projection, Complete life Table in main features and construction.

Index Number - Definition and applications, price relative and quality or volume relatives, link and chain relative, problems involved in the computation of index number, use of average, sample aggregative and weighted average methods, Laspeyres, Paasches, Fisher's index number, Time and Factor reversal test of index number, consumer price index.

Unit-6 : Statistical Inference :

Point estimation: properties of estimators, mean square and minimum mean square error estimator, unbiasedness and minimum variance unbiased estimator, Cramer-Rao lower bound, amount of information, consistency of estimators and sufficient conditions for consistency, relative efficiency of an estimator, asymptotic efficiency, sufficiency, factorization theorem (without proof), concept of complete sufficient statistics, Rao-Blackwell theorem. Completeness and sufficiency, Lehman Scheffe theorem, one parameter exponential family and its completeness, Cramer-Rao inequality, Best linear unbiased estimator.

Methods of estimation: moments, maximum likelihood, minimum chi-square, least square with examples, BAN and CAN estimators, point estimates of measures of location, dispersion, regression, correlation and other useful parameters.

Concepts of confidence interval and confidence coefficient, confidence intervals for the parameters of univariate normal, two independent normal distributions and exponential distributions.

Statistical hypotheses, critical region, size and power of a test, most powerful test, randomized and non-randomized test, Neyman-Pearson lemma and its applications,

...t powerful unbiased test, power likelihood ratio test and its applications, functions of UMP with simple illustration.

Applications of χ^2, t, F and z distributions in tests of significance.

Likelihood ratio test, Unbiased test, Neyman Pearson Lemma for randomized tests, randomized test for binomial and Poisson distribution, χ^2 –test of goodness of fit, Test of equality of several variances, Significance test for correlation coefficient.

Unit-7 : Sampling Theory :

Concept of population and sample, need for sampling, complete enumeration versus sampling, Basic concepts in sampling, sampling and Non-sampling errors.

Simple random sampling with and without replacement, estimation of population mean, population proportion and their standard errors. Stratified random sampling, proportional and optimum allocation, comparison with simple random sampling for fixed sample size. Post stratification, Gain impression due to stratification.

Ratio, product and regression methods of estimation, estimation of population mean, evaluation of bias and variance to the first order of approximation, comparison with simple random sampling.

Systematic sampling, Cluster sampling with equal clusters, two stage sampling.

Unequal probability sampling: PPSWR/PPSWOR methods of sample selection (including cumulative total method and Lahiri's scheme). Comparison of SRSWR and PPSWR schemes. Ordered estimators of Des Raj and Murthy (for n=2). Construction of unordered estimators from ordered estimators. Horvitz Thompson's estimator of a finite population total/mean. Expression for variance of Horvitz Thompson's estimator and unbiased estimator of the variance, Yates-Grundy modification.

Non-sampling errors and their types with special reference to non-response problems. Hansen and Hurwitz estimator for population mean. Concept of randomized response and some well-known randomized response techniques for estimation of sensitive characteristics.

Unit-8 : Linear Estimation and Design of Experiments :

General linear model, assumptions, estimation of parameters by least squares, estimable functions, error and estimation space, Gauss-Markov theorem.

Distribution of quadratic form and its application in analysis of variance model, Estimable linear hypothesis, generalized F and t tests.

One-way ANOVA, two-way ANOVA with single observation per cell and equal number of observations per cell, Tukey's test.

Design of Experiment : Randomization, Replication, Local Control, Completely Randomized Design (CRD),Randomized Block Design (RBD), Latin square design (LSD), Missing plot technique.

General block design and its information matrix (C). Criteria for connectedness, balanced and orthogonality, Intra-block analysis, Incomplete block design: Balanced Incomplete Block Design (BIBD).

General factorial experiments, factorial effects, study of 2^n factorial experiments in randomized blocks, complete and partial confounding, construction of confounded factorial experiments, split plot experiment.

Unit-9 : Multivariate Analysis :

Multivariate normal distribution (MND), Moment generating function and Characteristic function, marginal and conditional distributions, multiple and partial correlation coefficients for MND.

Maximum likelihood estimators of the mean vector and covariance matrix, Distribution of sample mean vector, null distribution of sample correlation coefficient, simple multiple and partial correlation coefficients and their null sampling distributions, distribution of sample regression coefficient.

Null distribution of Hotelling's T^2 statistic, Application in tests for mean vector of one and more multivariate normal populations and for equality of the components of a mean vector in a multivariate normal population and their applications, Mahalanobis' D^2, Wishart distribution and its properties.

Classification and discrimination procedures for discrimination between two multivariate normal distributions, populations-sample discriminant function, and test associated with discriminant functions, probabilities of misclassification and their estimation, classification into more than two multivariate normal populations, Fisher-Behrens Problem.

Unit-10 : Advance Probability Theory :

Classes of sets, fields, sigma fields, minimal sigma field, Borel sigma field, sequence of sets, *lim sup* and *lim inf* of a sequence of sets, measure, probability measure, properties of measure, Lebesgue and Lebesgue-Stieltjes measures.

Measurable functions, random variables, sequence of random variables, Integration of a measurable function with respect to a measure, monotone convergence theorem, Fatou's lemma, dominated convergence theorem. Characteristic function, uniqueness theorem, Convergence in distribution, Convergence in probability, almost sure convergence.

Decomposition of distribution functions in purely discrete, absolutely continuous and singular components. Holder's inequality, Minkowski inequality, Lyapunov, Kolmogorov's inequality.

Weak (WLLN) and Strong (SLLN) law of Large Numbers, Khintchine Theorem and Kolmogorov Strong Law of Large Numbers, Borel zero-one Law, Borel-Cantelli lemma.

Weak and complete convergence of sequence of distribution functions, Weak compactness Theorem, Helly-Bray Lemma & Theorem Characteristics functions, Inversion Theorem

One dimensional Central Limit Theorems: Lindberg- Levy for *i.i.d.* random variables.