## Section - B (26 - 100) (Mathematics)

26. The limit of the function

$$\frac{(\operatorname{Re} z)(\operatorname{Im} z)}{|z|}$$

at $z = 0$

(A) does not exist

(B) is zero

(C) exists but non-zero

(D) is infinite

27. The set of points

$\{z \in \mathbb{C} : \operatorname{Re} z = |z - 1|\}$ is a

(A) Straight Line

(B) Circle

(C) Parabola

(D) Ellipse

28. The hyperbolic function $\cosh z$ :

(A) is never zero in $\mathbb{C}$

(B) is zero for all $z = 2k\pi i$, $k \in \mathbb{Z}$

(C) is zero for all $z = \frac{1}{2}(2k + 1)\pi i$, $k \in \mathbb{Z}$

(D) is zero for all $z = 2k\pi$, $k \in \mathbb{Z}$

29. The radius of convergence of the power series $\sum (-1)^n n^{2024} z^n$ is :

(A) 0

(B) 1

(C) 2024

(D) $\infty$

30. The value of the contour integral

$\int_r \operatorname{Im}(z^2)\,dz$

over the circle $r : |z| = 1$ is :

(A) 0

(B) $2\pi i$

(C) $\pi i$

(D) $2\pi$

31. If $r$ is the circle centred at the origin with radius 1, then the contour integral $\int_r |z|^4 (\operatorname{Re} z)\,dz$ equals

(A) 0

(B) $\pi$

(C) $\pi i$

(D) $2\pi i$

32. If $r$ is the circle of radius one centered at the origin, then

$$\int_r \frac{\operatorname{Re} z}{z - \frac{1}{2}}\,dz \text{ equals}$$

(A) $\frac{1}{2}\pi i$

(B) $\pi i$

(C) $2\pi i$

(D) 0

33. The residue of $(\cot z)/z^2$ at the origin is :

(A) 0

(B) 1

(C) $\frac{1}{2}$

(D) $-\frac{1}{3}$

34. The singularity of the function

$$F(z) = \frac{(z-1)^3 \cos \pi z}{(2z-1)(z^2+1)^6 \sin^3 \pi z}$$

at the point $z = \frac{1}{2}$ is :

(A) Removable singularity

(B) Pole of order 1

(C) Non-isolated singularity

(D) Essential singularity

78. Given below are two statements

**Statement I** The partial differential equation

$$yu\frac{\partial^2 u}{\partial x^2} + \frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2} + u\frac{\partial u}{\partial y} + e^x u + e^y = 0$$

is semi - linear

**Statement II** The partial differential equation

$$y\frac{\partial^2 u}{\partial x^2} + x\frac{\partial^2 u}{\partial y^2} = 0$$

is of hyperbolic type at the point $(x, y) = (1, -2)$

In light of the above statement, choose the most appropriate answer from the codes given below

(A) Both Statement I and II are correct

(B) Both Statement I and II are incorrect

(C) Statement I is correct and Statement II is incorrect

(D) Statement I is incorrect and Statement II is correct

79. Which one of the following identities hold for every solution $u = u(x, y)$ of the equation

$$\frac{\partial^2 u}{\partial x^2} + 3\frac{\partial^2 u}{\partial x \partial y} + 2\frac{\partial^2 u}{\partial y^2} = 0 ?$$

(A) $u(0, 0) + u(1, 2) + u(-1, 0) + u(-2, -2) = 0$

(B) $u(0, 0) - u(1, 2) + u(-1, 0) + u(-2, -2) = 0$

(C) $u(0, 0) - u(1, 2) - u(-1, 0) + u(-2, -2) = 0$

(D) $u(0, 0) + u(1, 2) - u(-1, 0) - u(-2, -2) = 0$

80. Let $u = u(x, t)$ be the solution to the problem

$$\frac{\partial^2 u}{\partial t^2} - 16\frac{\partial^2 u}{\partial x^2} = 0, \; x \in \mathbb{R}, \; t > 0,$$

$$u(x, 0) = e^{\sin\left(\frac{x}{4}\right)}, \; x \in \mathbb{R}$$

$$\frac{\partial u}{\partial t}(x, 0) = 0, \; x \in \mathbb{R}$$

The smallest natural number $k$ such that $u(x + k\pi, t) = u(x, t)$ for all $x \in \mathbb{R}, t > 0$ is :

(A) 2

(B) 4

(C) 6

(D) 8

81. Let $\Omega = \{(x, y) \in \mathbb{R}^2 / x^2 + y^2 < 4\}$ which one of the following triples $(\alpha, \beta, \gamma) \in \mathbb{R}^3$ is such that the Neumann problem

$\Delta u = 0$ in $\Omega$

$\partial_n u = \alpha x^2 + \beta y + \gamma$ for $x^2 + y^2 = 4$

does NOT admit a solution $u \in C^2(\bar{\Omega})$?

(A) $(\alpha, \beta, \gamma) = (-1, 3, 2)$

(B) $(\alpha, \beta, \gamma) = (-2, 4, 4)$

(C) $(\alpha, \beta, \gamma) = (3, -2, -1)$

(D) $(\alpha, \beta, \gamma) = (5, 3, -10)$

12. Let $f: \mathbb{R}^2 \to \mathbb{R}$ be defined by $f(0,0) = 0$ and $f(x,y) = \frac{x^2 y}{x^4 + y^2}$ for $(x,y) \neq (0,0)$

Which one of the following is false for the function $f$?

(A) $f$ is continuous at the origin

(B) The partial derivatives of $f$ exist at the origin

(C) The directional derivatives of $f$ in any direction exist at the origin

(D) The function $f$ is not differentiable at the origin

13. Let $f: \mathbb{R}^2 \to \mathbb{R}^2$ be defined by the equation

$f(r, \theta) = (r\cos\theta, r\sin\theta)$

The largest value of '$b$' for which the function f is one-to-one on $A = (0, 1) \times (0, b)$ in the $(r, \theta)$ -plane is :

(A) $\frac{\pi}{2}$

(B) $\pi$

(C) $2\pi$

(D) $\frac{\pi}{4}$

14. The subset

$A = \{(r\cos\frac{2k\pi}{2025}, r\sin\frac{2k\pi}{2025}) \mid k = 0, 1, \ldots, 2024;\ r \geq 0\}$ of $\mathbb{R}^2$ is :

(A) compact in $\mathbb{R}^2$

(B) dense in $\mathbb{R}^2$

(C) has 2025 components

(D) both connected and path connected

15. For a subset $A$ of metric space $X$, let

$d(x, A) = \inf\{d(x, a) \mid a \in A\}$

Then $d(x, A) = 0$ if and only if :

(A) $x \in A$

(B) $x \in A^C$, the complement of $A$

(C) $x \in \bar{A}$, the closure of $A$

(D) $x \in \partial A$ the boundary of $A$

87 Let $p(x)$ denote the interpolation polynomial for the data

| $x$ | 0 | 1 | 3 | −1 |
|---|---|---|---|---|
| $y$ | 1 | 1 | −11 | 5 |

Then, the value of $p(2)$ is

(A) −1

(B) −2

(C) −5

(D) −8

88 Given below are two statements

**Statement I** : $x^4 + x^2 - 2$ is the interpolating polynomial for the data

| $x$ | −1 | 0 | 1 |
|---|---|---|---|
| $y$ | 0 | −2 | 0 |

**Statement II** : If $p(x)$ is the polynomial of least degree satisfying $p(1) = 1$, $p'(1) = 2$, $p(2) = 3$, $p'(2) = 4$, $p''(2) = 5$, then the degree of $p(x)$ is 5 :

In light of the above statement, choose the most appropriate answer from the codes given below :

(A) Both Statement I and II are correct

(B) Both Statement I and II are Incorrect

(C) Statement I is correct and Statement II is incorrect

(D) Statement I is incorrect and Statement II is correct

89 For $h > 0$, the formula

$$D_h^* f(x) := \frac{f(x+h) - f(x-h)}{2h}$$ is

used to approximate the derivative $f'(x)$ of a function $f$. Then $D_h^* f(x) - f'(x)$ is equal to

(A) $\frac{h}{2} f''(\eta)$ for some $\eta \in (x-h, x+h)$

(B) $\frac{h^3}{6} f''(\eta)$ for some $\eta \in (x-h, x+h)$

(C) $\frac{h^2}{6} f^{(3)}(\eta)$ for some $\eta \in (x-h, x+h)$

(D) $\frac{h^3}{24} f^{(4)}(\eta)$ for some $\eta \in (x-h, x+h)$

90 If the quadrature formula

$$\int_0^{\pi} f(x)dx = Af(0) + Bf(\pi)$$

is exact for all functions of the form $a + b\cos x$, where $a, b \in \mathbb{R}$, then the value of $3A+5B$ is :

(A) 0

(B) $2\pi$

(C) $4\pi$

(D) $8\pi$

82. Let $u = u(x_1, x_2)$ be the solution to the Dirichlet boundary value problem

$\Delta u = 0$ in $\{(x_1, x_2) \in \mathbb{R}^2 : x_1^2 + x_2^2 < 1\}$,
$u(x_1, x_2) = 6 - 4x_1 + 4x_2$ for $x_1^2 + x_2^2 = 1$,

then the value of $u\left(\frac{1}{2}, -\frac{1}{2}\right)$ is :

(A) 1 (B) 2
(C) 3 (D) 4

83. The Gauss-Seidel iterative sequence for the system

$$\begin{pmatrix} 1 & 2 \\ 2 & \alpha \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \text{ where } \alpha \in \mathbb{R},$$

converges for every $\begin{pmatrix} x_1^{(0)} \\ x_2^{(0)} \end{pmatrix} \in \mathbb{R}^2$ if and only if :

(A) $\alpha \neq 4$ (B) $|\alpha| < 4$
(C) $|\alpha| \neq 4$ (D) $|\alpha| > 4$

84. For $\alpha, \beta \in \mathbb{R}$ consider the system

$$\begin{pmatrix} \alpha & 3 & 1 \\ 2 & \beta & 3 \\ 1 & 0 & 2 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 2 \\ 5 \\ -2 \end{pmatrix}$$

For which one of the following choices of $(\alpha, \beta \in \mathbb{R}^2)$, Gaussian elimination method can not be applied to find a solution of the given system?

(A) $(\alpha, \beta) = (2, 5)$
(B) $(\alpha, \beta) = (-1, 2)$
(C) $(\alpha, \beta) = (-1, -3)$
(D) $(\alpha, \beta) = (1, 2)$

85. Given below are two statements

**Statement I** : There exists a function $f : \mathbb{R} \to \mathbb{R}$ such that the iterative sequence of Newton-Raphson method for the solution of $f(x) = 0$ satisfy $x_0 = 0$, $x_n = n$ for $n \geq 1$

**Statement II** : The rate of convergence of Newton-Raphson method is 2 when applied to the solution of the nonlinear equation $x^2 - 6x + 9$.

In light of the above statement, choose the most appropriate answer from the codes given below :

(A) Both Statement I and II are correct
(B) Both Statement I and II are Incorrect
(C) Statement I is correct and Statement II is incorrect
(D) Statement I is incorrect and Statement II is correct

86. For a $2 \times 2$ matrix $A$, Let $\|A\|_\infty$ denote the matrix norm of $A$. Which is subordinate to the vector norm defined by

$$\left\|\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}\right\|_\infty = \max\{|x_1|, |x_2|\} \text{ for } \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \in \mathbb{R}^2$$

With respect to the given matrix norm, the condition number of the matrix $\begin{pmatrix} 5 & -2 \\ 1 & 1 \end{pmatrix}$ is :

(A) 6 (B) 7
(C) 36 (D) 49

15. The number of zeros of the analytic function $2 + z^3 - z^n$ in the open upper half-plane $\operatorname{Im} z > 0$ is

(A) 0 (B) 1
(C) 2 (D) 3

16. The function $\frac{\pi \cot \pi z}{z^2}$ has

(A) Removable singularity at $z = 0$
(B) Single pole at $z = 0$ with residue $\frac{\pi^2}{3}$
(C) Double pole at $z = 0$ with residue $\frac{\pi^2}{3}$
(D) Triple pole at $z = 0$ with residue $\frac{\pi^2}{3}$

17. The Laurent expansion of cosec $z$ is

(A) $\frac{1}{z} + \frac{z}{3!} + \dots$ valid for $0 < |z| < \pi$
(B) $\frac{1}{z} - \frac{z}{3} + \dots$ valid for $0 < |z| < 2\pi$
(C) $\frac{1}{z} + \frac{z^2}{2!} + \dots$ valid for $0 < |z| < \frac{\pi}{2}$
(D) $\frac{1}{z} - \frac{z}{3!} + \dots$ valid for $0 < |z| < \pi$

18. The Laurent expansion of $1/z$ valid in $|z-1| < 1$

(A) $\sum_{n=0}^{\infty} (-1)^n (z-1)^n$
(B) $\sum_{n=0}^{\infty} (z-1)^n$
(C) $\sum_{n=0}^{\infty} (-1)^n (z-1)^{-n}$
(D) $\sum_{n=0}^{\infty} (z-1)^{-n}$

39. The mapping $w = \sin z$ maps

(A) The lines parallel to the imaginary axes to ellipses
(B) The lines parallel to the imaginary axes to parabolas
(C) The lines parallel to the real axes to parabolas
(D) The lines parallel to the real axes to ellipses

40. The circle that will be mapped to a straight line by the transformation $w = (1+z)/(1-z)$ is

(A) $|z| = 2$
(B) $|z-2| = 1$
(C) $|z+1| = 1$
(D) $|z-1| = 1$

41. Given 4 different elements $a, b, c, d$ the number of 3-combinations with repetition (such as $aaa$, $aab$) is

(A) 4 (B) 64
(C) 20 (D) 24

20. The system of equations

$x + 2y + 3z = b_1$, $2x - y + z = b_2$,

$4x + 3y + 7z = b_3$

is consistant if $(b_1, b_2, b_3)$ equals

(A) (6, 7, 7)

(B) (0, 1, 2)

(C) (1, 1, 3)

(D) (1, 2, 0)

21. The coordinate vector relative to the ordered basis {(1, 2, 1), (2, 9, 0), (3, 3, 4)} of the vector $v = (5, -1, 9)$ is

(A) (1, 3, −2)

(B) (−1, −3, 2)

(C) (1, 3, −2)

(D) (−1, 3, 2)

22. The eigenvalues of the adjoint matrix

of $\begin{pmatrix} 2 & 0 & -1 \\ 0 & 2 & 0 \\ -1 & 0 & 2 \end{pmatrix}$ are

(A) 1, 2, 3

(B) $1, \frac{1}{2}, \frac{1}{3}$

(C) $\frac{1}{6}, \frac{1}{3}, \frac{1}{2}$

(D) 2, 3, 6

23. If $A$ is a matrix of order 3 with 1, 2, −1 as its eigenvalues, the adjoint of $A$ is given by

(A) $A^2 + 2A + I$

(B) $A^2 - 2A - I$

(C) $A^2 - 2A + I$

(D) $A^2 + 2A - I$

24. The canonical form of the quadratic form is $2xy + 2yz + 2xz$ is

(A) $2u^2 + v^2 + w^2$

(B) $2u^2 - v^2 + w^2$

(C) $2u^2 - v^2 - w^2$

(D) $-2u^2 + v^2 + w^2$

25. The quadratic form

$x^2 - y^2 - z^2 + 2xy + 2yz + 2xz$ is

(A) Positive definite

(B) Positive semi-definite

(C) Negative semi-definite

(D) Indefinite

73. Let $S$ denotes the set of all $\lambda \in \mathbb{R}$ such that the boundary value problem

$$\frac{d^2 y}{dx^2} - \lambda y = 0$$

$$\frac{dy}{dx}(0) = \frac{dy}{dx}(\ell) = 0$$

has a non-zero solution, where $\ell > 0$

Then the set $S$ is given by

(A) $\{n^2 \pi^2 / n \in \mathbb{N}\}$

(B) $\{0\} \cup \{\frac{n^2 \pi^2}{\ell} / n \in \mathbb{N}\}$

(C) $\{-\frac{n^2 \pi^2}{\ell} / n \in \mathbb{N}\}$

(D) $\{0\} \cup \{-n^2 \pi^2 / n \in \mathbb{N}\}$

74. Which one of the following partial differential equations is obtained on eliminating the arbitrary function of satisfying the relation :

$f(x^2 + y^2, z - xy) = 0$ ?

(A) $y\frac{\partial u}{\partial x} + x\frac{\partial u}{\partial y} - x^2 - y^2 = 0$

(B) $y\frac{\partial u}{\partial x} + x\frac{\partial u}{\partial y} + x^2 + y^2 = 0$

(C) $y\frac{\partial u}{\partial x} - x\frac{\partial u}{\partial y} - x^2 + y^2 = 0$

(D) $y\frac{\partial u}{\partial x} - x\frac{\partial u}{\partial y} + x^2 - y^2 = 0$

75. Consider the Cauchy problem for the equation

$$2\frac{\partial u}{\partial x} + 3\frac{\partial u}{\partial y} = u$$

Where the Cauchy data is prescribed on a line ($\ell$) in the $xy$-plane

Which one of the following lines has the property that there exists a Cauchy data for which the Cauchy problem does NOT have a solution?

(A) $\ell : 2x + 3y = 0$

(B) $\ell : 2x - 3y = 1$

(C) $\ell : 3x - 2y = 2$

(D) $\ell : 3x + 2y = 3$

76. The general solution of

$$y^2 u \frac{\partial u}{\partial x} + x u^2 \frac{\partial u}{\partial y} = -x y^2$$

given by Lagrange's method, in terms of an arbitrary differentiable function $f$, is given by

(A) $f(x^2 + u^2, 3x^2 u - 2y^3) = 0$

(B) $f(x^2 + u^2, y^3 + u^3) = 0$

(C) $f(3x^2 u - 2y^3, y^3 + u^3) = 0$

(D) $f(x^2 + u^2, 3x^2 u - 2y^3) = 0$

77. Let $u := u(x, y)$ be the solution to the Cauchy problem

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = u^2,$$

$$u(x, 0) = \frac{1}{x}, \; x > 0$$

Then, the value of $u\left(1, -\frac{3}{2}\right)$ is :

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$

(C) $\frac{3}{4}$ (D) $\frac{3}{2}$

7. The infinite series

$$\sum_{n=0}^{\infty} \frac{x}{(1+|\sin x|)^n}, \quad x \in (-\pi, \pi)$$

converges to a function which is :

(A) Continuous everywhere

(B) Discontinuous at one point

(C) Discontinuous at countably infinite number of points

(D) Discontinuous everywhere

8. Let $f : [a,b] \to \mathbb{R}$ be the identity function given by $f(x) = x$ and $P_n$ be the partition $P_n = \{a < a+h < a+2h < \ldots < a+nh = b\}$. Then lower Riemann sum $L(f, P_n)$ equals :

(A) $\dfrac{b^2-a^2}{2}$

(B) $\dfrac{b^2-a^2}{2} - \dfrac{(b-a)^2}{2n}$

(C) $\dfrac{b^2-a^2}{2} - \dfrac{(b-a)^2}{n}$

(D) $\dfrac{b^2-a^2}{2} + \dfrac{(b-a)^2}{2n}$

9. The Cantor set is :

(A) Lebesgue measurable and countable

(B) Uncountable but not Lebesgue measurable

(C) Uncountable with Lebesgue measure zero

(D) Uncountable with non-zero Lebesgue measure

10. Which one of the following is false for Lebesgue outer measure $m^*$?

(A) $m^*([a,b]) = b-a$

(B) For any set $A$ and $\in > 0$, there is an open set $U \supseteq A$ such that $m^*(U) \leq m^*(A) + \in$

(C) $m^*(A) = 0 \Rightarrow m^*(A \cup B) = m^*(B)$ for any set B

(D) For any sequence of sets $\{E_i\}$, $\sum_{i=1}^{\infty} m^*(E_i) \leq m^*\left(\bigcup_{i=1}^{\infty} E_i\right)$

11. Consider the function $f : \mathbb{R}^2 \to \mathbb{R}$ given by $f(x,y) = \dfrac{xy}{x^2+y^2}$ for $(x,y) \neq (0,0)$ and $f(0,0) = 0$, then at $(x,y) = (0,0)$ :

(A) The partial derivatives of $f$ with respect to $x$ does not exist

(B) The partial derivatives of $f$ with respect to $y$ does not exist.

(C) The partial derivatives exist but $f$ is not differentiable

(D) $f$ is differentiable

63 Given below are two statements

Let be the space of $\mathbb{R}$ real numbers with cofinite topology

**Statement I** Every subset of $\mathbb{R}$ is a compact set in $\mathbb{R}$.

**Statement II** Only proper closed subset of $\mathbb{R}$ are compact sets

In light of the above statement, choose the most appropriate answer from the codes given below

(A) Both Statement I and II are correct

(B) Both Statement I and II are incorrect

(C) Statement I is correct and Statement II is incorrect

(D) Statement I is incorrect and Statement II is correct

64 Given below are two statements :

Let $f: \mathbb{R} \to \mathbb{R}^N$ be defined by $f(t) = (t, t, ...)$, $\mathbb{R}^N$ is the countable product of with itself

**Statement I**: The function of $f$ is continuous when $\mathbb{R}^N$ is given product topology.

**Statement II** The function f is not continuous with respect to box topology on $\mathbb{R}^N$.

In light of the above statement, choose the most appropriate answer from the codes given below :

(A) Both Statement I and II are correct

(B) Both Statement I and II are incorrect

(C) Statement I is correct and Statement II is incorrect

(D) Statement I is incorrect and Statement II is correct

65 Given below are two statements

Let $X$ be an infinite space with discrete topology

**Statement I** A closed and bounded subset of $X$ is compact

**Statement II** No finite subset of $X$ is compact

In light of the above statement, choose the most appropriate answer from the codes given below

(A) Both Statement I and II are correct

(B) Both Statement I and II are incorrect

(C) Statement I is correct and Statement II is incorrect

(D) Statement I is incorrect and Statement II is correct

66. Let $\varphi:(0,\infty)\to\mathbb{R}$ be the solution to the initial value problem

$$\frac{dy}{dx} = \frac{y^2 - 2xy + 2x^2}{x^2},$$

$y(1)=3/2$

Then the value of $\varphi(3)$ is

(A) 3.5

(B) 3.75

(C) 4.25

(D) 4.5

42. The number of integers from 1 to 1000 that are not divisible by 2, 3, or 5 is
(A) 720
(B) 734
(C) 246
(D) 266

43. Ten straight lines in a plane are such that no two of which are parallel and no three of which pass through a point. Then these ten lines cut the plane into $n$ regions, where $n$ is
(A) 90
(B) 100
(C) 110
(D) 112

44. The equation $x^2 - y^2 = 1002$ has :
(A) Finitely many integer solutions
(B) Unique integer solution
(C) Finitely many solutions in rational numbers
(D) No solution in rational numbers

45. The number of permutations of 1, 2, 3, ..., 8, 9 in which all even integers are in their natural positions and none of the odd integers are in their natural position is equal to :
(A) 24
(B) 40
(C) 44
(D) 60

46. Let $G$ be a group of order 40. Which one of the following is false for $G$?
(A) If $K$ is a normal subgroup of order 5 and $H$ any subgroup of order 8, then $G = HK$
(B) If $K$ is a normal subgroup of order 5 and $H$ any normal subgroup of order 8, then $G = H \times K$
(C) If $G$ has only one subgroup of order 8, it is normal
(D) $G$ has either 2 or five subgroups of order 8

47. The ring $\mathbb{Z}_{36}$ has
(A) $\langle 2 \rangle$ as the only maximal ideal
(B) $\langle 6 \rangle$ as the only maximal ideal
(C) both $\langle 2 \rangle$ and $\langle 3 \rangle$ as maximal ideals
(D) $\langle 2 \rangle, \langle 3 \rangle, \langle 4 \rangle, \langle 9 \rangle, \langle 6 \rangle$ as maximal ideals

48. The polynomial $x^2 + 1$ is reducible over :
(A) $\mathbb{Z}_3$
(B) $\mathbb{Z}_4$
(C) $\mathbb{Z}_5$
(D) $\mathbb{Z}_7$

49. Let ED, PID, UFD stand for Euclidean domain, principal ideal domain, unique factorization domain. Which one of the following is false ?
(A) Every PID is ED
(B) Every PID is UFD
(C) Every ED is PID
(D) Every ED is UFD

67\. Given below are two statements

Consider the initial value problem (IVP)

$$\frac{dy}{dx} = 3y^{2/3}$$

$$y(0) = y_0$$

**Statement I** : There exists a $y_0 \in \mathbb{R}$ such that the IVP has a unique solution

**Statement II** : $\forall\, y_0 \in \mathbb{R}$, the IVP has a solution defined for every $x \in \mathbb{R}$

In light of the above statement, choose the most appropriate answer from the codes given below :

(A) Both Statement I and II are correct

(B) Both Statement I and II are incorrect

(C) Statement I is correct and Statement II is incorrect

(D) Statement I is incorrect and Statement II is correct

68\. Let $y_1(x)$, $y_2(x)$ be the solution to the system of ordinary differential equations

$$\frac{dy_1}{dx} = y_2 + \frac{1}{2}\sin 2x$$

$$\frac{dy_2}{dx} = -y_1 - \cos 2x$$

Satisfying $y_1(0) = 1$, $y_2(0) = 0$

then the value of

$$y_1\left(\frac{\pi}{4}\right) + y_2\left(\frac{\pi}{4}\right) \text{ is}$$

(A) 1

(B) -0.5

(C) 0.25

(D) 1

69\. Given below are two statements

Regarding the initial value problem

(IVP) $\frac{dy}{dx} = \sin y^2$, $y(0) = y_0$

**Statement I** : $\forall y_0 \in \mathbb{R}$, every solution of the IVP is bounded

**Statement II** : $\exists y_0 \in \mathbb{R}$ such that the IVP does not have a solution that is defined for every $x \in \mathbb{R}$

In light of the above statement, choose the most appropriate answer from the codes given below :

(A) Both Statement I and II are correct

(B) Both Statement I and II are Incorrect

(C) Statement I is correct and Statement II is incorrect

(D) Statement I is incorrect and Statement II is correct

50. The number of elements in $\mathbb{Z}_3[x]/\langle x^3+2x+2\rangle$ is

(A) 9 (B) 27

(C) 10 (D) 81

51. Which one of the following is not an integral domain?

(A) $\mathbb{Z}[i]$ of Gaussian integers

(B) $\mathbb{Z}_p$ of integers modulo a prime $p$

(C) $\mathbb{Z} \oplus \mathbb{Z}$

(D) $\mathbb{Z}[x]$ of all polynomials with integer coefficients

52. The symmetric group $S_n$ is not solvable if $n$ equals

(A) 2

(B) 3

(C) 4

(D) 5

53. Let $G = \mathbb{Z}_8 \oplus \mathbb{Z}_4$ and let $H = \langle (2,2) \rangle$ be a cyclic subgroup of $G$. Then $G/H$ is isomorphic to :

(A) $\mathbb{Z}_4$

(B) $\mathbb{Z}_4 \oplus \mathbb{Z}_2$

(C) $\mathbb{Z}_2 \oplus \mathbb{Z}_2 \oplus \mathbb{Z}_2$

(D) Both $\mathbb{Z}_4$ and $\mathbb{Z}_2 \oplus \mathbb{Z}_2 \oplus \mathbb{Z}_2$

54. From among the four given choices, the smallest positive integer that leaves a reminder 2 when divided by 3, a reminder 3 when divided by 5 and a reminder 2 when divided by 7 is

(A) 238 (B) 448

(C) 548 (D) 643

55. Given below are two statements

**Statement I** The order of the permutation $(1, 4, 3, 2)(5, 6)$ is 4

**Statement II** The order of the permutation $(1, 2, 3)(1, 4, 5)$ is 5

In light of the above statement, choose the most appropriate answer from the codes given below

(A) Both Statement I and II are correct

(B) Both Statement I and II are Incorrect

(C) Statement I is correct and Statement II is incorrect

(D) Statement I is incorrect and Statement II is correct

56. Let $Y$ be the subset $[0, 1) \cup \{2\}$ of $\mathbb{R}$. Which one of the following is false?

(A) The set $\{2\}$ is open in the subspace topology on $Y$

(B) The set $\{2\}$ is open in the order topology on $Y$

(C) Any basis element for the order topology that contains 2 is of the form $\{x \mid x \in Y \text{ and } a < x \le 2\}$

(D) The order topology and the subspace on $Y$ are different

57. If $X = \{a, b, c\}$ and $\tau = \{\phi, X, \{b\}, \{a,b\}, \{b,c\}\}$ a topology on $X$, the sequence $\langle x_n \rangle$ with $x_n = b$ for all $n$ converges to

(A) $b$ only

(B) All elements of $X$

(C) None of the elements of $X$

(D) $a$ and $c$ only

91. Let $y = y(x)$ be the solution to $\frac{dy}{dx} = x^2 + y^2$, $y(0) = 1$ using the value $y_1$ obtained by approximating the value of $y(0.1)$ by Euler's method. The approximate value of $y(0.2)$ obtained by the modified Euler method (Also known as Euler's mid-point method) is :

(A) 1.122

(B) 1.244

(C) 1.322

(D) 1.344

92. Let $C^1[0,1]$ denotes the space of all functions which are continuous along with their first order derivatives on the interval [0, 1]. Define $J : C^1[0,1] \to \mathbb{R}$ by

$$J[y(x)] = \int_0^1 \sqrt{y}\left(1 + \left(\frac{dy}{dx}\right)^2\right)dx$$

Then the second variation of the functional $J$ at the constant function $y_0(x) = 1$, in the usual notation, is given by :

(A) $\int_0^1 \{-\frac{1}{4}(\delta y)^2 + 2(\frac{d}{dx}(\delta y))^2\}dx$

(B) $\int_0^1 \{\frac{1}{2}(\delta y)^2 + 8(\frac{d}{dx}(\delta y))^2\}dx$

(C) $\int_0^1 \{(\delta y)^2 - (\delta y)(\frac{d}{dx}(\delta y)) + \frac{d}{dx}(\delta y))^2\}dx$

(D) $\int_0^1 \{(\delta y)^2 + 2(\delta y)(\frac{d}{dx}(\delta y)) + 2(\frac{d}{dx}(\delta y))^2\}dx$

93. Let $y = y(x)$ be a stationary function (also known as extremal) of the functional

$$J[y(x)] = \int_0^{\pi/2}\left\{(y')^2 - 2y\sin x\right\}dx$$

satisfying the conditions $y(0) = 1$, $y\left(\frac{\pi}{2}\right) = 2$

Then the value of $y(x)$ at $x = \frac{\pi}{6}$ is :

(A) $\frac{5}{2}$

(B) $\frac{\pi}{3} + \frac{3}{2}$

(C) $\frac{3}{2}$

(D) $\frac{\pi}{6} + \frac{2}{3}$

**Note :** This paper contains 175 multiple choice questions of 2 marks each in THREE (3) Sections. Attempt all the questions either from Section - A and Section - B only OR from Section - A and Section - C only. The OMR sheet with questions attempted from both Sections viz. Section - B and Section - C will not be evaluated. Number of questions Section wise: Section - A : Q No. 1 to 25, Section - B : Q No. 26 to 100, Section C : Q No. 101 to 175

## Section - A (1 - 25)

1. The set $S$ is uncountable if $S$ consists of all

   (A) Integers, both positive and negative

   (B) Algebraic numbers

   (C) Positive transcendental numbers

   (D) Polynomial functions with integer coefficients

2. For $A, B \in P(X)$, the power set of a set $X$, define $A \oplus B = (A - B) \cup (B - A)$. Which one of the following is false?

   (A) $A \oplus A = \phi$

   (B) $A \oplus \phi = A$

   (C) $A \oplus (X - A) = \phi$

   (D) $(A \oplus B) \oplus C = A \oplus (B \oplus C)$

3. The value of the limit

$$\lim_{n \to \infty} \left[ \frac{(kn)!}{(n!)^k} \right]^{1/n}$$

   for any given natural number $k$, is :

   (A) $k$

   (B) $3^k$

   (C) $k^3$

   (D) $k^k$

4. The function $f : \mathbb{R} \to \mathbb{R}$ defined by

$$f(x) = \begin{cases} \sin \pi x & x \in \mathbb{Q} \\ -\sin \pi x & x \in \mathbb{R} - \mathbb{Q} \end{cases}$$

   is :

   (A) Continuous only at $x = 0$

   (B) Continuous only at $x = 1$

   (C) Continuous at all $x \in \mathbb{Z}$

   (D) Continuous at all $x \in \mathbb{Q}$

5. The Lagrange's mean-value theorem for the function $f : [-1, 3] \to \mathbb{R}$ defined by $f(x) = (x - 1)(x + 3)$ is satisfied with $C$ given by

   (A) $-1$

   (B) $0$

   (C) $1$

   (D) $2$

6. If $S$ and $T$ are subsets of $\mathbb{R}$, then

   (A) $\text{int}\,(S \cup T) = \text{int}\, S \cup \text{int}\, T$

   (B) $\text{int}\, S \cup \text{int}\, T \subseteq \text{int}\,(S \cup T)$

   (C) $\text{int}\,(S \cap T) \neq \text{int}\, S \cap \text{int}\, T$

   (D) $\text{int}\,(S \cup T) \subset \text{int}\, S \cup \text{int}\, T$

58. Which one of the following is false for a continuous function from a topological space $X$ to another topological space $Y$?

(A) For every $A \subseteq X$, $f(\overline{A}) \subseteq \overline{f(A)}$

(B) For every open $U \subseteq X$, $f(U)$ is open

(C) For each $x \in X$ and each neighbourhood $V$ of $f(x)$, there is a neighbourhood $U$ of $x$ st $f(U) \subseteq V$

(D) For every closed set $B$ of $Y$, the set $f^{-1}(B)$ is closed in $X$

59. Let $\mathbb{R}$ denotes the set of real numbers in its usual topology and let $\mathbb{R}_\ell$ denotes the same in the lower limit topology. Then which one of the following is true for the identity mapping $f : \mathbb{R} \to \mathbb{R}_\ell$?

(A) Both $f$ and $f^{-1}$ are continuous

(B) $f$ is continuous, $f^{-1}$ is not continuous

(C) $f$ is not continuous, $f^{-1}$ is continuous

(D) Neither $f$ nor $f^{-1}$ is continuous

60. The closure of the set

$\{(x, \sin(\frac{1}{x})) \mid 0 < x \le 1\} \subseteq \mathbb{R}^2$ is

(A) Connected but not path connected

(B) Connected as well as path connected

(C) Path connected but not connected

(D) Neither connected nor path connected

61. If $\mathbb{R}_\ell$ is $\mathbb{R}$ with lower limit topology and $\mathbb{R}_\ell^2$ is the Sorgenfrey plane, then which one of the following is false?

(A) $\mathbb{R}_\ell$ is not normal

(B) $\mathbb{R}_\ell^2$ is not normal

(C) $\mathbb{R}_\ell^2$ is regular

(D) $\mathbb{R}^J$ is not normal if $J$ is uncountable

62. Let the space $\mathbb{R}_\ell$ be $\mathbb{R}$ with lower limit topology. Which one of the following is false for $\mathbb{R}_\ell$?

(A) $\mathbb{R}_\ell$ satisfies the first countability axiom

(B) $\mathbb{R}_\ell$ satisfies the second countability axiom

(C) $\mathbb{R}_\ell$ is a Lindelöf space

(D) $\mathbb{R}_\ell$ is separable

94 The Euler-Lagrange equation for the functional

$$J[u(x,y)] = \iint_{\Omega} \left\{ x^2\left(\frac{\partial u}{\partial x}\right)^2 + y^2\left(\frac{\partial u}{\partial y}\right)^2 + u^2 \right\} dx\,dy$$

where $\Omega$ is a bounded domain in $\mathbb{R}^2$ is

(A) $2x\frac{\partial u}{\partial x} + x^2\frac{\partial^2 u}{\partial x^2} + 2y\frac{\partial u}{\partial y} + y^2\frac{\partial^2 u}{\partial y^2} = u$

(B) $x\left(\frac{\partial u}{\partial x}\right)^2 + 2x^2\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial x^2} + y\left(\frac{\partial u}{\partial y}\right)^2 + 2y^2\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y^2} = u$

(C) $x^2\frac{\partial^2 u}{\partial x^2} + y^2\frac{\partial^2 u}{\partial y^2} = u$

(D) $x\left(\frac{\partial u}{\partial x}\right)^2 + x\frac{\partial^2 u}{\partial x^2} + y\left(\frac{\partial u}{\partial y}\right)^2 + y^2\frac{\partial^2 u}{\partial y^2} = u$

95 Which of the following statements is correct regarding the extremals of the functional

$$J[y(x)] = \int_0^{\pi/2} \left\{ \frac{1}{2}(y')^2 + yy' + y' - 2y^2 \right\} dx$$

where the end-value $y(0)$ and $y\left(\frac{\pi}{2}\right)$ are free?

(A) Exactly one extremal

(B) Exactly two extremals

(C) No Extremals

(D) Infinitely many extremals

96 Given below are two statements

Consider the ordinary differential equation (ODE) $y'' + \lambda y = g(x)$, $x \in (0,1)$ and $g : [0, 1] \to \mathbb{R}$ is a continuous function.

**Statement I:** If $y := y(x)$ is a solution to the ODE such that $y(0)=0$, $\frac{dy}{dx}(0) = 1$, then $y$ satisfies an integral equation of Volterra type.

**Statement II:** If $y := y(x)$ is a solution to the ODE such that $y(0)=0$, $y(1)=1$, then $y$ satisfies an integral equation of Volterra type.

In light of the above statements, choose the most appropriate answer from the codes given below :

(A) Both Statement I and II are correct

(B) Both Statement I and II are incorrect

(C) Statement I is correct and Statement II is incorrect

(D) Statement I is incorrect and Statement II is correct

97 Let $\varphi = \varphi(x)$ be the solution to the integral equation

$$\varphi(x) = 2e^{x} + e^{x}\cos x + \int_{0}^{x} \frac{2+\cos x}{2+\cos t}\varphi(t)\,dt\,,$$

$x > 0$, then the value of $\varphi(\pi)$ is

(A) $\pi e^{\pi}$

(B) $(1+\pi)e^{\pi}$

(C) $\pi(2+\pi)e^{\pi}$

(D) $(1+\pi)(2+\pi)e^{\pi}$

98 Let $\varphi := \varphi(x)$ be the solution to the integral equation

$$\varphi(x) = 2\int_{0}^{\pi/2}(1-\cos 2x)\varphi(t)dt + 2x\,,$$

$x \in \left[0, \frac{\pi}{2}\right]$, then the value of $\varphi\left(\frac{\pi}{2}\right)$ is

(A) $\frac{\pi}{1-\pi}$

(B) $\frac{1-\pi}{2-\pi}$

(C) $\frac{\pi(2-\pi)}{1-\pi}$

(D) $\frac{\pi(1-\pi)}{2-\pi}$

99 For the homogeneous integral equation

$$\varphi(x) - \lambda\int_{-1}^{1}(5xt^{3} + 4t)\varphi(t)\,dt = 0$$

which one of the following is a characteristic number?

(A) $\frac{1}{2}$

(B) $\frac{4}{5}$

(C) $\frac{5}{4}$

(D) 2

100 For a one dimensional system with the Hamiltonian

$$H = \frac{p^{2}}{2} - \frac{1}{2q^{2}}$$

which one of the following is a constant of motion?

(A) $\frac{pq}{2} - Ht$

(B) $\frac{pq}{2} - Ht^{2}$

(C) $\frac{pq}{2} - H\frac{t^{2}}{2}$

(D) $\frac{pq}{2} - Ht^{3}$

16. The set $S$ spans the vector space $P_2$ of all polynomials of degrees $\leq 2$ if

(A) $S = \{1+x+x^2, -1-x, 2+2x+x^2\}$

(B) $S = \{x+x^2, x-x^2, 1+x, 1-x\}$

(C) $S = \{1+x+x^2, -1-x^2, 2+x+2x^2\}$

(D) $S = \{1, x+x^2, 1+x+x^2, 1-x-x^2\}$

17. Consider $\mathbb{R}^2$ with inner product defined by

$$\langle u, v\rangle = \frac{1}{9}u_1v_1 + \frac{1}{4}u_2v_2$$

Where $u = (u_1, u_2)$, $v = (v_1, v_2)$, then $\|u\| = 1$ represents a :

(A) Square

(B) Circle

(C) Ellipse

(D) Rhombus

18. The Gram-Schmidt orthogonalization process transforms the basis $\{(1, 1, 1), (0, 1, 1), (0, 0, 1)\}$ of $\mathbb{R}^3$ into an orthogonal basis given by $\{v_1, v_2, v_3\}$ where $v_1 = (1, 1, 1)$, $v_2 = \left(-\frac{2}{3}, \frac{1}{3}, \frac{1}{3}\right)$ and $v_3$ given by :

(A) $v_3 = \left(-\frac{1}{2}, \frac{1}{2}, 0\right)$

(B) $v_3 = \left(\frac{1}{3}, -\frac{2}{3}, 0\right)$

(C) $v_3 = \left(0, 0, \frac{1}{3}\right)$

(D) $v_3 = \left(0, -\frac{1}{2}, \frac{1}{2}\right)$

19. Let $P_n$ be the vector space of all polynomilas of degree less than or equal to $n$ with standard basis $\{1, x, \ldots, x^n\}$. The matrix for the transformation $T : P_1 \to P_2$ defined by $T(P(x)) = xP(x)$ is

(A) $\begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}$

(B) $\begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}$

(C) $\begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$

(D) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$