A

19221 **120 MINUTES**

1. The function $f: \mathbb{R} \to \mathbb{R}$ defined by $f(x) = x|x|$ is
   A) injective but not surjective B) surjective but not injective
   C) bijective D) neither injective nor bijective

2. The set of values of m such that the roots of the equation $3x^2 + 2x + m(m-1) = 0$ are of opposite signs is
   A) $(0,1)$ B) $[0,1]$ C) $[0, 1)$ D) $(0, \infty)$

3. The foot of the perpendicular drawn from the point $(-2,-2)$ to the line $x + y = 2$ is
   A) $(-2, 0)$ B) $(1, 1)$ C) $(0, -2)$ D) $(-1, -1)$

4. If the lines $3x + 4y + 17 = 0$ and $6x + 8y + 9 = 0$ are tangents to the same circle, then the radius of the circle is
   A) 5 B) $\frac{5}{2}$ C) $\frac{5}{4}$ D) $\frac{5}{3}$

5. An equilateral triangle is inscribed in the parabola $y^2 = 4x$ with one of the vertices at the vertex of the parabola. Then the perimeter of the triangle is
   A) 12 B) $12\sqrt{3}$ C) $8\sqrt{3}$ D) $24\sqrt{3}$

6. The equation of the sphere passing through origin and having radius 1 and centre on the positive $z -$ axis is
   A) $x^2 + y^2 + z^2 - 2z = 0$ B) $x^2 + y^2 + z^2 - 2x - 2y = 0$
   C) $x^2 + y^2 + z^2 - 2y = 0$ D) $x^2 + y^2 + z^2 - 2z = 1$

7. If $f(x) = \begin{cases} \frac{e^{\frac{1}{x}}-1}{e^{\frac{1}{x}}+1}, & if x \neq 0 \\ 0 & if x = 0 \end{cases}$.

   Then $\lim_{x \to 0} f(x)$ is
   A) 0 B) 1 C) -1 D) Does not exist

8. Suppose $(x) = x - [x]$ where $[x]$ denotes the greatest integer less than or equal to x, then $\lim_{n \to 0} \frac{(x)+(2x)+\cdots+(nx)}{n^2}$ is
   A) x B) $\frac{x}{2}$ C) $\frac{x}{3}$ D) $\frac{x}{4}$

9. A bag contains 3black, 3 white and 1 red balls. Three balls are drawn one after the other without replacement. The probability that the third ball is red is:

A) $\frac{5}{7}$ B) $\frac{2}{7}$ C) $\frac{3}{7}$ D) $\frac{1}{7}$

10. A problem in statistics is given to three students whose chances of solving it individually are $\frac{1}{3}, \frac{1}{4}, \frac{1}{5}$ respectively. The probability that the problems was solved by exactly one of them is

A) $\frac{11}{30}$ B) $\frac{12}{30}$ C) $\frac{13}{30}$ D) $\frac{7}{30}$

11. If $f(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right), & x \neq 0 \\ 0 & x = 0 \end{cases}$

then,

A) f is not continues at $x = 0$

B) f is continues everywhere but it is not differentiable at at $x = 0$

C) f is differentiable everywhere but its first derivative $f'(x)$ is not continuous

D) f is infinitely differentiable

12. If $\int \frac{x\tan^{-1}x}{\sqrt{1+x^2}} dx = \int x\tan^{-1}x + g(x)$, then

A) $f(x) = \sqrt{1+x^2}, g(x) = -\log(x+\sqrt{1+x^2}) + c$

B) $f(x) = \frac{1}{\sqrt{1+x^2}}, g(x) = -\log(x-\sqrt{1+x^2}) + c$

C) $f(x) = -\sqrt{1+x^2}, g(x) = \log(x+\sqrt{1+x^2}) + c$

D) $f(x) = \sqrt{1+x^2}, g(x) = -\log(x-\sqrt{1+x^2}) + c$

13. Let (x_n) and (y_n) be two real sequences with $x_n = \frac{2^n}{n!}$, $y_n = n^{\frac{1}{n}}$, $n \geq 1$

A) (x_n) converges to 0 and (y_n) converges to 1

B) (x_n) converges to 1 and (y_n) converges to 1

C) (x_n) converges to 1 and (y_n) converges to 0

D) (x_n) converges to 0 and (y_n) converges to 0

14. The value of the integral $\int_0^3 [x] d(x^2) dx$, where $[x]$ denotes the greatest integer not greater than x is

A) $\frac{9}{2}$ B) 13 C) $\frac{55}{3}$ D) 10

15. Let f be the function defined by $f(x) = \begin{cases} x \sin\left(\frac{\pi}{x}\right), & 0 < x \leq 2 \\ 0, & x = 0 \end{cases}$. Then

A) f is continuous but not of bounded variation

B) f is bounded and not of bounded variation

C) f is neither continuous nor of bounded variation

D) f is bounded, continuous and of bounded variation

16. Let $\langle f_n \rangle$ be a sequence of non-negative measurable functions that converges almost everywhere to a function f and if $f_n \leq f$, for all $n, then$

A) $\int f < \lim \int f_n$ B) $\int f = \lim \int f_n$

C) $\int f = 0$ D) $\int f > \lim \int f_n$

17. Suppose $= f(z) = \sum_{n=1}^{\infty} \frac{(-1)^n}{n} z^{n(n+1)}$. Then

A) $f(z)$ converges for $|z| < 1$ and diverges at $z=1$

B) $f(z)$ converges for $|z| \leq 1$

C) $f(z)$ converges for $|z| > 1$

D) $f(z)$ converges for all z

18. Let x be a connected open subset of $\mathbb{C}$ and $f: X \rightarrow \mathbb{C}$ be an analytic function. Then which of the following is true?

A) If $f(z)$ is real for all $z \in X$, then f is a constant function

B) If $f(z)$ is real for all $z \in X$, then $f \equiv 0$

C) If $f(z)$ is purely imaginary for all $z \in X$, then $f \equiv 0$

D) If $f(z)$ is real for all $z \in X$, then $f(z) > 0$ for all z

19. Let Cbe the circle $\{z: |z| = 2\}$. The value of $\int_c \frac{e^{iz}}{(z^2+10)\sin z} dz$ is

A) $2\pi i$ B) πi C) $-2\pi i$ D) 0

20. If $z_i, i = 1, 2, \ldots .5$ are the 5^{th} roots of unity, then $\sum_{i=1}^{5} \frac{1}{z_i}$ is

A) 5 B) $\frac{1}{5}$ C) 0 D) 5i

21. Let G be an infinite cyclic group. Then the number of automorphisms on G is

A) 1 B) 2 C) 3 D) 4

22. The largest order of an element in $\mathbb{Z}_4 \times \mathbb{Z}_8 \times \mathbb{Z}_3$ is

A) 96 B) 48 C) 24 D) 12

23. The inverse of the permutation (1, 2, 3) (4, 7, 8) in S_8 is

A) (1, 3, 2) (4, 7, 8) B) (1, 2, 3) (4, 7, 8)
C) (1, 3, 2) (7, 4, 8) D) (1, 3, 2) (4, 8, 7)

24. The number of solutions of $x^2 + 8x - 3 = 0$ in $\mathbb{Z}_{12}$ is

A) 0 B) 1 C) 2 D) 12

25. Which of the following is true?

A) The fields $\mathbb{R}$ and $\mathbb{C}$ are isomorphic
B) $2\mathbb{Z}$ and $3\mathbb{Z}$ are isomorphic rings
C) There is only one ring homomorphism from $\mathbb{Z}$ to $\mathbb{Z}$
D) There is only one group homomorphism from $\mathbb{Z}$ to $\mathbb{Z}$

26. The number of zero divisors of $\mathbb{Z}_{360}$ is

A) 360 B) 180 C) 96 D) 48

27. Suppose $f(x) = x^4 + 3x^2 + 2$. Which of the following is true?

A) $f(x)$ is irreducible over $\mathbb{R}$
B) $f(x)$ is irreducible over $\mathbb{Q}$ but reducible over $\mathbb{R}$
C) $f(x)$ is irreducible over $\mathbb{C}$
D) $f(x)$ is reducible over $\mathbb{Q}$ and $\mathbb{R}$

28. Which of the following is true?

A) π is algebraic over $\mathbb{R}$
B) π is algebraic over $\mathbb{Q}$
C) $\sqrt{2}$ is transcendental over $\mathbb{R}$
D) $\sqrt{2} + \sqrt{3}$ is transcendental over $\mathbb{R}$

29. The degree of $\mathbb{Q}(\sqrt{2}, \sqrt{3})$ over $\mathbb{Q}$ is

A) 1 B) 2 C) 4 D) 8

30. Let R be a ring with unity and M be an ideal containing a unit, then

A) M is a proper ideal
B) $M = R$
C) R/M *is a field,* if R is commutative ring
D) M is the trivial ideal $\{0\}$

31. Let A and B be two square matrices of same order defined over $\mathbb{R}$ and B be non-singular. Then,

A) rank (AB) = rank (B) B) rank (BA) = rank (B)
C) rank (AB) = rank (A) D) rank $(AB) \leq$ rank (B)

32. The value of a for which the system of equations $x + y + z = 1, x - y + 2z = 0,$ $2x + 3z = a$ has infinitely many solutions is

A) 1 B) 2 C) 3 D) 0

33. Let $T: \mathbb{R}^3 \to \mathbb{R}^3$ be defined by $T(x,y,z) = (x, x+y, x+y+z)$. The null space of T is

A) $\{(x,y,z): x-y=0\}$ B) $\{(x,y,z): x+y=0\}$

C) $\{(x,y,z): y-z=0\}$ D) $\{(0, 0, 0)\}$

34. Suppose $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$. Then

A) A and B have different characteristic equations but the same minimal polynomials

B) A and B are diagonalizable

C) A is diagonalizable but B is not

D) B is diagonalizable but A is not

35. Let $A \in M_3(\mathbb{R})$, the set of all 3 x 3 matrices over $\mathbb{R}$, be a symmetric matrix. Which of the following can be the characteristic values of A?

A) $1, 0, 1$ B) $1, 1+i, 1-i$

C) $1, i, -i$ D) $0, 1, i$

36. If the characteristic roots of $\in M_3(\mathbb{R})$ are $1, \omega, \omega^2$, the cube roots of unity, then the characteristic roots of A^{-1} are

A) $1, \omega, \omega^2$ B) $2, 1+\omega, 1+\omega^2$

C) $1, 1-\omega, 1-\omega^2$, D) $1, 1, 1$

37. Let $T: \mathbb{R}^2 \to \mathbb{R}^2$ be defined by $T(x,y) = (y, 2x)$. $S = \{(x,y): x^2+y^2 = 1\}$.Then $T(S)$ is

A) $\{(x,y): x^2+y^2 = 1\}$ B) $\{(x,y): 2x^2+y^2 = 1\}$

C) $\{(x,y): 2x^2+y^2 = 2\}$ D) $\{(x,y): x^2+2y^2 = 2\}$

38. If n is an odd integer, n^2 is of the form

A) $8k+1, k \in \mathbb{Z}$ B) $8k-1, k \in \mathbb{Z}$

C) $16k+1, k \in \mathbb{Z}$ D) $16k-1, k \in \mathbb{Z}$

39. The highest integer value of n such that 3^n divides 1749600 is

A) 5 B) 6 C) 7 D) 8

40. Which of the following is not true if n is an integer greater than 1?

A) $1+2+\cdots+(n-1) \equiv 0 \ (mod\ n)$

B) $1^2+2^2+\cdots+(n-1)^2 \equiv 0 \ (mod\ n)$

C) $1^3+2^3+\cdots+(n-1)^3 \equiv 0 \ (mod\ n)$

D) $1^2-2^2+\cdots+(2n-1)^2-(2n)^2 \equiv 0 \ (mod\ n)$

41. The orthogonal trajectories of the family of curve $y^2 = 4a(x+a)$ is

A) $y^2 = 4a(x+a)^2$ B) $x^2 = 4a(y+a)$

C) $y = 4a(x+a)$ D) $y^2 = 4a(x+a)$

42. If W is the Wronskian of the differential equation $y'' + y = 0$, then

A) $|W| = 1$ B) $|W| > 1$ C) $|W| < 1$ D) $W = 0$

43. Let $\{P_n(x)\}$ be the sequence of Legendre polynomials

A) $\int_{-1}^{1} P_n(x)P_m(x)dx = 1$ if $m = n$

B) $\int_{-1}^{1} P_n(x)P_m(x)dx = \frac{1}{2n+1}$ if $m \neq n$

C) $\int_{-1}^{1} P_n(x)P_m(x)dx = 0$ if $m = n$

D) $\int_{-1}^{1} P_n(x)P_m(x)dx = \frac{2}{2n+1}$ if $m = n$

44. Let $J_p(x)$ denote the Bessel function. Which of the following is true?

A) $\frac{d}{dx}\left(x^p J_p(x)\right) = x^{p-1} J_{p-1}(x)$

B) $\frac{d}{dx}\left(x^p J_p(x)\right) = x^p J_{p-1}(x)$

C) $\frac{d}{dx}\left(x^{-p} J_p(x)\right) = x^{-p} J_{p+1}(x)$

D) $\frac{d}{dx}\left(x^{-p} J_p(x)\right) = x^{-p} J_{p-1}(x)$

45. The solution of the partial differential equation $(z-y)\frac{\partial u}{\partial x} + (x-z)\frac{\partial u}{\partial y} = y - x$ is

A) $x^2 + y^2 + z^2 = f(x + y - z)$
B) $x^2 + y^2 + z^2 = f(x - y + z)$
C) $x^2 + y^2 + z^2 = f(x + y + z)$
D) $x^2 + y^2 - z^2 = f(x + y + z)$

46. The solution of the partial differential equation $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{\partial u}{\partial x} \cdot \frac{\partial u}{\partial y}$

A) $az = (ax + y)^2 + c$ B) $z^2 = (ax + y)^2 + c$
C) $z = (x - y)^2 + c$ D) $2az = (ax + y)^2 + c$

47. The equation $2u_{xx} + 4xy\, u_{xy} - u_{yy} = 0$ is
A) Parabolic B) Elliptic
C) Hyperbolic D) Parabolic only in the region where $y \geq 0$

48. Suppose $X = \{0, 1, 2, 3\}$, $\tau_1 = \{X, \emptyset, \{0\}, \{3\}, \{0, 3\}\}$, $\tau_2 = \{X, \emptyset, \{3\}, \{0, 1, 3\}\}$. Let the map $f : (X, \tau_1) \rightarrow (X, \tau_2)$ be defined as $f(x) = x$, Then
A) f is continuous but not open.
B) f is open and not continuous.
C) f is both open and continuous.
D) f is neither open nor continuous.

49. Let $X = C[0,1]$, the collection of all continuous function on the closed interval [0, 1]. Consider the metrics ρ and d defined by, $\rho(f,g) = sup_{x\in X}|f(x) - g(x)|$ and $d(f,g) = \int_0^1 |f(x) - g(x)|dx$. Then

A) The topology generated by ρ is stronger than the topology generated by d
B) The topology generated by d is stronger than the topology generated by ρ
C) Both d and ρ generate the same topology
D) The largest topology contained in the intersection of the topologies generated by ρ and d is the trivial topology.

50. Which of the following is not a topological property?

A) Compactness B) Connectedness
C) Completeness D) First countability

51. Let $(X, \|,\|)$ be a normed linear space over $\mathbb{C}$. Then which of the following is not true?

A) Every closed subset of X is compact.
B) If $\{x \in X : \| x \| \leq 1\}$ is compact, then X is finite dimensional.
C) If every closed and bounded subset of X is compact then X is finite dimensional.
D) If $\{x \in X : \| x \| \leq 1\}$ is compact, then X is infinite dimensional.

52. Let $X = \mathbb{R}^2$ with norm$\|,\|$ defined by $\| (x,y) \| = \sqrt{x^2 + y^2}$ and $f: X \to X$ with $f(x,y) = x + y$. Then $\| f \|$ is
A) 1 B) $\sqrt{2}$ C) 2 D) $\sqrt{3}$

53. Suppose $X = L^2[0,1]$ with inner product defined by $\langle f(t), g(t) \rangle = \int_0^1 f(t)\overline{g(t)}dt$. If $f(t) = t$ and $g(t) = 1 - t$, then $\| f - g \|$, where $\|,\|$ is the norm induced by the inner product.

A) 1 B) $\frac{1}{3}$ C) $\frac{1}{6}$ D) $\frac{1}{2}$

54. Suppose $X = L^2[-\pi, -\pi]$. For each $n \in \mathbb{N}$, let (f_n) be a sequence of functions defined on $[-\pi, \pi]$, by $f_n(t) = e^{int}$. Then,

A) $\{f_n\}$ is an orthonormal set in X
B) $\{f_n\}$ is an orthogonal set but not orthonormal in X
C) $\{f_n\}$ is neither orthonormal set nor orthogonal in X
D) $\{f_n\}$ is both orthonormal and orthogonal in X

55. Let P be the projection map defined on a normed linear space X and I the identity map. Which of the following is not true?
A) $I - P$ is also a projection
B) Range of P is the zero space of $I - P$
C) Zero space of P is the zero space of $I - P$
D) There can be a non-zero element in the intersection of Range space of P and zero space of P

56. The area of the triangle whose vertices are the third roots of unity in the complex plane is

A) $\frac{\sqrt{3}}{4}$ B) $\frac{\sqrt{3}}{2}$ C) $\sqrt{3}$ D) $\frac{3\sqrt{3}}{4}$

57. The area of the region $\{(x, y) \in \mathbb{R}^2; x^2 \leq y \leq 1 - x^2\}$ is

A) $\frac{2}{3}$ B) $\frac{2\sqrt{2}}{3}$ C) $\frac{2}{\sqrt{3}}$ D) $2\sqrt{3}$

58. From a group of 7 men and 6 women, five persons are to be selected to form a committee so that at least 3 men are there on the committee. In how many ways can it be done?

A) 525 B) 756 C) 221 D) 635

59. Consider the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$ and $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$. Then

A) both series are conditionally convergent

B) both series are absolutely convergent

C) the first series is conditionally convergent and the second series is absolutely convergent

D) the first series is absolutely convergent and the second series is conditionally convergent

60. The number of zeros of $z^5 + 3z^2 + 1$ in $|z| < 1$, counted with multiplicity is

A) 0 B) 1 C) 2 D) 3

61. The harmonic conjugate of the function $u(x, y) = x^3 - 3xy^2$ is

A) $3x^2y - y^3$ B) $3xy^2$ C) $y^3 - 3xy^2$ D) $3xy^2 - y^3$

62. The total number of subgroups of a cyclic group of order 24 is

A) 8 B) 6 C) 4 D) 2

63. If the matrix $\begin{pmatrix} 2 & 1 & 0 \\ 0 & 2 & 2 \\ 1 & 1 & 2 \end{pmatrix}$ is invertible in $\mathbb{Z}/n\mathbb{Z}$, then

(i) $\gcd(2, n) = 1$ (ii) $\gcd(3, n) = 1$ (iii) $\gcd(6, n) = 1$

Choose the correct statement(s):

A) (i) only B) (ii) only

C) (i) and (ii) only D) (i), (ii) and (iii)

64. The number of subfields of a field with 2^8 elements is:

A) 1 B) 2 C) 4 D) 8

65. Let I, J be ideals in $\mathbb{Z}[x]$ generated by $x^3 + x^2 + x + 1$ and $x^3 - 2x^2 + x - 2$ respectively. Then $I + J$ is generated by the polynomial

A) $2x^3 - x^2 + 2x - 1$ B) $x + 1$

C) $x - 1$ D) $x^2 + 1$

66. Which of the following is necessarily an invertible matrix?

A) A nilpotent matrix B) An idempotent matrix

C) An orthogonal matrix D) A symmetric matrix

67. The rank of the matrix $\begin{pmatrix} 1 & 2 & 0 & 1 \\ 5 & 4 & 2 & 6 \\ 4 & 2 & 2 & 5 \end{pmatrix}$ is

A) 1 B) 2 C) 3 D) 4

68. Let A be an $n \times n$ matrix. Then $\det(5A) = \cdots$

A) $5\det(A)$ B) $5^n\det(A)$ C) $5^2\det(A)$ D) $n^5\det(A)$

69. The matrix of change of basis from the standard basis (e_1, e_2) of $\mathbb{R}^2$ to $(e_1 + e_2, e_1 - e_2)$ is

A) $\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$ B) $\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$ C) $\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$ D) $\frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$

70. Which of the following maps are linear transformations from $\mathbb{R}^2$ to $\mathbb{R}^3$?

(i) $f(x, y) = (x + 2, y + 2, 2)$ (ii) $f(x, y) = (x + y, x - y, 0)$

(iii) $f(x, y) = (x, y, xy)$ (iv) $f(x, y) = (x + y, x - y, 2y)$

A) (i) and (ii) B) (ii) and (iii) C) (ii) and (iv) D) (iii) and (iv)

71. Let T be a linear operator on $\mathbb{C}^n$, $n > 1$ such that every non-zero vector of $\mathbb{C}^n$ is an eigen vector of T. Then

A) all eigen values of T are distinct

B) all eigen values of T are real

C) all eigen values of T are equal

D) T must be the zero matrix

72. Let V be the the vector space of all polynomials with degree $\leq$ n and let T be any linear functional on V. Then dim(Ker T) is

A) 1 B) $n-1$ C) n D) $n+1$

73. The system $x \equiv 1(\text{mod}6)$, $x \equiv 1(\text{mod}4)$ has

A) exactly one solution, modulo12

B) exactly two solutions, modulo12

C) exactly six solutions, modulo12

D) no solution

74. Find a collection of linearly independent solutions of $\frac{d^4y}{dx^4} - \frac{d^2y}{dx^2} = 0$.

A) $\{1, x, e^x, e^{-x}\}$ B) $\{1, x, e^{-x}, xe^{-x}\}$

C) $\{1, x, e^x, xe^x\}$ D) $\{1, x, e^x, xe^{-x}\}$

75. The envelope of the 1-parameter family $(x-a)^2 + (y-2a)^2 + z^2 = 1$ is

A) $z = \pm 1$ B) $(2x-y)^2 + 5z^2 = 5$

C) $(2x-y)^2 = 1$ D) $x^2 + y^2 = 1$

76. Which of the following is a wave equation?

A) $u_{tt} = u_{xx}$ B) $u_t = u_x$ C) $u_t = u_{xx}$ D) $u_{tt} = u_x$

77. Which of the following is is not a metric on ℝ?

A) $d(x,y) = \sqrt{|x-y|}$ B) $d(x,y) = |e^x - e^y|$

C) $d(x,y) = |x^3 - y^3|$ D) $d(x,y) = \frac{|\,|x|-|y|\,|}{1+|xy|}$

78. Let (X, d) be a metric space. For $A, B \subseteq X$, define $d(A,B) == \inf\{d(x,y);\ x \in A, y \in B\}$. Choose the correct statement(s).

(i) If A and B are disjoint, then $d(A,B) > 0$

(ii) If A and B are closed and disjoint, then $d(A,B) > 0$

(iii) If A and B are compact and disjoint, then $d(A,B) > 0$

A) (i),(ii) and (iii) B) (ii) and (iii) only

C) (iii) only D) none of these

79. Which of the following is not true for a non-trivial linear vector space X?

A) There is a norm on X

B) There is a norm on X which induces the discrete metric

C) The sum of two norms on X is a norm on X

D) Any metric induced by a norm on X is unbounded

80. Which of the following space is a Hilbert space?

A) l^∞ space B) l^1 space C) l^2 space D) l^4 space