A

19731 **120 MINUTES**

1. Which of the following statement is/are true?
   a) Lebesgue measure of any line of finite length in R^k is zero
   b) Lebesgue measure of any line of infinite length in R^k is zero

   A) (a) only B) (b) only
   C) Both (a) and (b) D) Neither (a) nor (b)

2. If f and g are measurable functions then which of the following is not measurable?
   A) $f + g$ B) fg C) $f - g$ D) None of these

3. A and B be p x p matrices. Which of the following equals $trace\ ((AB)^2)$
   A) $trace\ (A^2B^2)$ B) $trace\ (AB^2A)$ C) $trace\ ((BA)^2)$ D) $(trace(AB))^2$

4. Let A is an m x n matrix. If A^- is the generalized inverse of A, then
   A) $AA^- = I$ B) $AA^-A = A$
   C) A^- is unique D) rank (A^-) < rank (A)

5. Let $T : V -> W$ be a linear transformation with $dim(V) = dim(W)$.
   If $\{b_1, b_2, .. b_n\}$ is a basis for V then:
   A) $T(b_1), T(b_2), .., T(b_n)$ is a basis for W.
   B) $T(b_1), T(b_2), .., T(b_n)$ are linearly independent.
   C) $T(b_1), T(b_2), .., T(b_n)$ form a basis for W if and only if T is one to one.
   D) None of the above

6. If A and B are two matrices, then
   A) rank (AB) = max (rank(A), rank(B))
   B) rank $(AB) \geq$ max (rank(A),rank(B))
   C) rank$(AB) \geq$ min (rank(A),rank(B))
   D) rank$(AB) \leq$ min (rank(A),rank(B))

7. Let U and W be finite dimensional subspaces of a vector space V. Then
   A) $dim(U+W) = dim(U)+dim(W)$
   B) $dim(U+W) > dim(U)+dim(W)$
   C) $dim(U+W) = min(dim(U), dim(W))$
   D) $dim(U+W) < dim(U)+dim(W)$

8. If f is continuous from R to R, then which of the following is/are correct?
   (a) $|f|$ is continuous.
   (b) $\sqrt{f}$ is continuous

   A) (a) alone B) (b) alone
   C) Both (a) and (b) D) Neither (a) nor (b)

9. If $f(x) = |x|$, then f is:
A) f is continuous but not differentiable.
B) f is neither continuous nor differentiable
C) f is continuous and differentiable.
D) f is not continuous but differentiable.

10. The $\lim_{x\to 0} \frac{\cos(x)-1}{x}$ is

A) 0 B) 1 C) -1 D) Does not exist

11. If A and B be two events, then which of the following is true?
A) $P(A \cap B) = P(A)P(B)$ B) $P(A \cap B) \geq 1 - P(A^c) - P(B^c)$
C) $P(A \cup B) > P(A)+P(B)$ D) $P(A \cap B) \leq P(A)+P(B) - 1$

12. Let A and B be two independent events. If $P(A) = \frac{1}{4}$ and $P(B) = \frac{1}{3}$, then $P(A^c/B^c)$ is
A) $\frac{1}{4}$ B) $\frac{9}{8}$ C) $\frac{1}{12}$ D) $\frac{3}{4}$

13. Suppose you have a coin with probability $\frac{1}{4}$ of getting a head. If you toss the coin twice independently, then what is the probability of getting at least one head?
A) $\frac{1}{4}$ B) $\frac{7}{16}$ C) $\frac{3}{16}$ D) $\frac{3}{8}$

14. A biased coin is tossed until a head appears for the first time. Let p be the possibility of a head, $0 < p < 1$. The probability that the number of tosses required is odd is:

A) $\left(\frac{1}{2}\right)^p$ B) $\frac{P}{2}$ C) $\frac{1}{2-P}$ D) P

15. If X follows *Binomial* (n, p) then $n - X$ follows:
A) *Binomial* (n, p) B) *Binomial* $(n, 1-p)$
C) *Binomial* $(2n, p)$ D) *Binomial* $(2n, 1-p)$

16. Let X be a non-negative random variable with distribution function F. Then $E(X)$ is:
A) $\int_0^\infty xF(x)dx$ B) $\int_0^\infty F(x)dx$

C) $\int_0^\infty x[1 - F(x)]dx$ D) $\int_0^\infty [1 - F(x)]dx$

17. Let X be a random variable. Then which of the following is not always a random variable?
A) $|X|$ B) X^2 C) $X^{\frac{1}{2}}$ D) $|X|^{\frac{1}{2}}$

18. If $X_1, X_2, \dots, X_n$ are iid N (0, 1) random variables, the limiting distribution of
$\sqrt{n}\, \frac{X_1 + X_2 +,\dots+X_n}{X_1{}^2 + X_2{}^2+,\dots X_n{}^2}$ *is*:

A) N (0, 1)
B) C (1, 0)
C) Student's t− distribution with 1 degree of freedom
D) Degenerate distribution

19. Let X be a continuous random variable which is symmetric about a. If $f(x)$ is the p.d.f of x, then

A) $f(x-a) = f(x+a)$ B) $f(x-a) = f(a-x)$

C) $f(x-a) = f(a)$ D) $f(x+a) = f(a-x)$

20. Let X be a random variable with m.g.f $M_X(t) = \left(1 - \frac{t}{\beta}\right)^{-\alpha}$ .Then $E(X)$ is:

A) $\alpha\beta$ B) $\frac{\alpha}{\beta}$ C) $-(\frac{\alpha}{\beta})$ D) $\frac{\beta}{\alpha}$

21. Let X and Y be two random variables. Which of the following is true?

A) $E[V(Y|X)] = V[E(Y|X)]+V(Y)$

B) $E[V(Y|X)]=V(Y|X)$

C) $E[V(Y|X)]=V(Y)-V[E(Y|X)]$

D) $E[V(Y|X)]=V(Y)|$

22. Let $Xn \xrightarrow{P} X$. Then $g(Xn) \xrightarrow{P} g(X)$ if

A) g is one to one B) g is onto

C) g is convex D) g is continuous

23. Let X be a continuous random variable with p.d.f $f(x) = ke^{-x/\theta};\ x \geq 0,\ \theta > 0$.
Then the median of X is:

A) 2θ B) e^{θ} C) $2\log\theta$ D) $\theta\log 2$

24. If $F(x)$ is the distribution function of a random variable X with mean 0 and variance σ^2, then

A) $F(x) \leq \frac{\sigma^2}{\sigma^2+x^2}, if\ x \leq 0$ B) $F(x) \geq \frac{\sigma^2}{\sigma^2+x^2}, if\ x \geq 0$

C) $F(x) \geq \frac{x^2}{\sigma^2+x^2}, if\ x \leq 0$ D) $F(x) \leq \frac{x^2}{\sigma^2+x^2}, if\ x \geq 0$

25. If $X \sim$ *Poisson* (λ) and the conditional distribution of $Y/X = x \sim$ *Binomial* (x,p), then Y follows

A) Poisson $\left(\frac{\lambda}{P}\right)$ B) *Binomial*$(x, p\lambda)$

C) *Poisson* (λp) D) Binomial $\left(x, \frac{\lambda}{P}\right)$

26. Let X and Y be two independent binomial random variables, having parameters (n_1, p) and (n_2, p), then the conditional distribution of X given $X+Y$ is

A) Poisson distribution B) Negative binomial distribution

C) Geometric distribution D) Hyper geometric distribution

27. If X $\sim U(0,1)$, then $-2log(1 - X)$ *follows:*

A) Chi-square with 2 d.f B) Exponential with mean 1

C) Exponential with mean $\frac{1}{2}$ D) Logarithmic uniform distribution

28. If X and Y are two independent *Gamma* (α_1, β) and *Gamma* (α_2, β) distributions, then the distribution of $\frac{X}{X+Y}$ is

A) *Beta* ($\alpha_1 + \alpha_2, \beta$) B) *Beta* ($\alpha_1, \alpha_2$)

C) $Beta\left(\frac{\alpha_1}{\alpha_1+\alpha_2}, \beta\right)$ D) *Beta* ($\alpha_1 + \alpha_2$)

29. If X follows Cauchy (1, 0), then $1/X$ follows

A) N (0, 1) B) Cauchy (1,0)

C) Inverse Cauchy (1, 0) D) Not exists

30. Let T follows t− *distribution with n d. f, then* T^2 follows

A) t − *distribution with* n^2 d.f

B) t − *distribution with 2n d. f*

C) *F* – *distribution* with (1,*n*) *d. f*

D) *F* – *distribution* with(1,2*n*) *d. f*

31. If X ~ $F(n_1, n_2)$, then $\left(1 + \frac{n_1}{n_2}X\right)^{-1}$ follows

A) $F\left(n_1, \frac{n_1}{n_2}\right)$ B) $Beta\left(\frac{n_2}{2}, \frac{n_1}{2}\right)$

C) *Beta* (n_1, n_2) D) None of the above

32. Let X~ *Binomial* (*n, p*), then *Cov* (*X, n*–*X*) is

A) *np* (1–*p*) B) $np^2(1-p)$ C) $-np(1-p)$ D) $np(1-p)^2$

33. If X ~ *Cauchy* (1, 0), then X^2 follows

A) *Gamma* (1,2) B) Chi-Square with 2d.f

C) *Cauchy* (1, *0*) D) *F*(1, 1)

34. Let $X_{(1)}, X_{(2)}, \ldots, X_{(n)}$ be the order statistics of a random sample of size n from $U(0,1)$ distribution, then the distribution of $X_{(r)}$, $1 \le r \le n$ is

A) *Beta* ($r, n-r+1$) B) *Beta* ($r+1, n-r$)

C) *Beta* (r, n) D) *Beta*($r+1, n-r+1$)

35. Let (X, Y) follows bivariate normal distribution with vector of parameters ($\mu_1, \mu_2, \sigma_1, \sigma_2, \rho$) then the regression of Y on X is:

A) $_1 + \rho\frac{\sigma_2}{\sigma_1}(y - \ _2)$ B) $_2 + \rho\frac{\sigma_1}{\sigma_2}(x - \ _1)$

C) $_1 + \rho\frac{\sigma_1}{\sigma_2}(y - \ _2)$ D) $_2 + \rho\frac{\sigma_2}{\sigma_1}(x - \ _1)$

36. Test statistic for testing the hypothesis $H_0 : \rho = 0$, where ρ is the population correlation coefficient follows:

A) t-distribution with n d.f B) t-distribution with $n-1$ d.f

C) t-distribution with n − 2 d.f D) t-distribution with 2n d.f

37. Let X_1 and X_2 be two independent $N(0,1)$ random variables, then $\frac{(X_1 - X_2)^2}{2}$

A) Gamma $\left(\frac{1}{2}, 1\right)$ B) Chi – *square with 2 d.f*

C) Gamma $\left(\frac{1}{2}, 2\right)$ D) Chi – *square with 4 d.f*

38. Mode of the Chi-square distribution with n d.f is:

A) n – 2 B) 2n C) n D) $\frac{n}{2}$

39. Let $X_1, X_2, \ldots, X_n$ be a random sample from $U(0,\theta)$. The UMVUE of θ is:

A) $\bar{X}$ B) $Max\ (X_1, X_2, \ldots, X_n)$

C) $Min\ (X_1, X_2, \ldots, X_n)$ D) $\frac{n+1}{n} Max\ (X_1, X_2, \ldots, X_n)$

40. Which of the following is a consistent estimate for θ in $C(\theta,\ 0)$:

A) Sample Mean B) Sample Variance

C) Sample Median D) None of these

41. Let X_1 and X_2 be a random sample from $N(\mu,\sigma^2)$. If $S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})^2$ then which among the following is/are true?

a) S^2 is UMVUE for σ^2

b) $Var(S^2)$ attains the Cramer-Rao Lower Bound

A) (a) alone B) (b) alone

C) Both (a) and (b) D) Neither (a) nor (b)

42. Let $X_1, X_2, \ldots, X_n$ be iid observations from $Gamma(\alpha,\ \beta)$. Which of the following is a sufficient statistics for $(\alpha,\ \beta)$?

A) $(\sum X_i, \prod X_i)$ B) $(\bar{X}, \prod X_i^{1/n})$

C) $(\bar{X},\ S^2)$ D) $(\sum X_i, \sum X_i^2)$

43. Based on a random sample of size n from $B(1,p)$ MLE of $p(1-p)$ *is*

A) $\bar{X}^2$ B) $\bar{X}(1-\bar{X})$ C) $\sum_{i=1}^{n} X_i(1-X_i)$ D) $\bar{X}$

44. Which of the following statements is / are true for MLE

(a) MLE is unique (b) MLE is not necessarily unbiased

A) (a) alone B) (b) alone

C) Both (a) and (b) D) Neither (a) nor (b)

45. Let T be a nunbiased estimator for θ then $E(T/S)$ *is the UMVUE for* θ *if*

A) S is sufficient for θ B) S is minimal sufficient for θ

C) S is complete sufficient for θ D) S is unbiased for θ

46. Neyman-Pearson lemma is used to find a most powerful test for testing
A) Simple hypothesis against composite alternative
B) Composite hypothesis against composite alternative
C) Composite hypothesis against simple alternative
D) Simple hypothesis against simple alternative

47. Rejecting a null hypothesis when it is true
A) Type I error B) Type II error C) No error D) Simple error

48. Which of the following is not belonging to exponential family but has an MLR property?
A) Cauchy $(0,\theta)$ B) Normal $(0,\sigma^2)$
C) Binomial $(0,\theta)$ D) Uniform $(0,\theta)$

49. Let λ be the likelihood ratio for testing the hypothesis $H_0 : \theta \in \Theta_0$ against $H_1 : \theta \in \Theta_1$. Then λ is a function of
A) Every consistent estimator for θ
B) Every sufficient statistic for θ
C) Every unbiased estimator for θ
D) None of these

50. The statistic H under the null hypothesis for Kruskal-Wallis test is approximately distributed
A) Chi-square distribution B) t-distribution
C) F-distribution D) Normal distribution

51. The distribution used in sign test is
A) Poisson distribution B) Uniform distribution
C) Binomial distribution D) Normal distribution

52. The Bayes estimate of a parameter θ under absolute error loss function is
A) Mean of the posterior distribution
B) Median of the posterior distribution
C) Mode of the posterior distribution
D) None of these

53. For the SPRT with strength (α, β) and boundary points A and B, A > B, which of the following is correct?
A) $A \geq \frac{1-\beta}{\alpha}$ and $B \leq \frac{1-\beta}{\alpha}$ B) $A \leq \frac{\beta}{1-\alpha}$ and $B \geq \frac{\alpha}{1-\beta}$

C) $A \leq \frac{1-\beta}{\alpha}$ and $B \geq \frac{\beta}{1-\alpha}$ D) $A \geq \frac{1-\alpha}{\beta}$ and $B \leq \frac{\alpha}{1-\beta}$

54. Let $X \sim Np(\theta, \Sigma)$. Then $X'\Sigma^{-1}X$ follows
A) $\chi^2_{(r)}$
B) Non-central $\chi^2_{(p)}$ with non-centrality parameter $\theta'\Sigma\theta$
C) $N_p(\theta, \Sigma)$
D) Non-central $\chi^2_{(p)}$ with non-centrality parameter $\theta'\Sigma^{-1}\theta$

55. If X ~ Np (0,Σ) then $X'BX$ follows

A) $\chi^2_{(p)}$ if and only if BΣ is idempotent of rank r
B) $\chi^2_{(r)}$ if and only if B is idempotent of rank r
C) $\chi^2_{(r)}$ if and only if BΣ is idempotent of rank r
D) $\chi^2_{(p)}$ if and only if B is idempotent of rank r

56. The multiple correlation coefficient lies between

A) 0 and ∞   B) -1 and 1   C) 0 and 1   D) -1 and 0

57. Wishart distribution is the multivariate analog of

A) Normal Distribution   B) F Distribution
C) t-Distribution   D) Chi-Square Distribution

58. Distribution of Hotelling's T^2 statistics is

A) F distribution   B) Whishart distribution
C) Chi-square distribution   D) t distribution

59. If S is the sample variance covariance matrix of a sample of size N from $Np(0,\Sigma)$ then NS follows:

A) $\chi^2_{(p)}$ B) $Wp\ (N-1, \Sigma)$ C) $Wp(N, \Sigma)$ D) $\chi^2_{(N-p)}$

60. The stationary distribution of the following transition probability matrix is:

$$\begin{bmatrix} 0 & 2/3 & 1/3 \\ 1/2 & 0 & 1/2 \\ 1/2 & 1/2 & 0 \end{bmatrix}$$

A) $\left(\frac{1}{3}, \frac{10}{27}, \frac{8}{27}\right)$ B) $\left(\frac{1}{3}, \frac{2}{5}, \frac{3}{5}\right)$

C) $\left(\frac{1}{3}, \frac{5}{9}, \frac{4}{27}\right)$ D) None of the above

61. Which of the following statement is/are true for a stochastic processes $\{X(t)\}$?

(a) A strictly stationary processes with $Var\ (X(t)) < \infty$ is also covariance stationary.

(b) A covariance stationary processes with $X(t)$ follows normal distribution is strictly stationary

A) (a) alone   B) (b) alone
C) Both(a)and (b)   D) Neither (a) nor (b)

62. Let N be a non-negative integer valued random variable with probability generating function $P_N(s)$. Let $X_n, n = 1, 2...,$ be a sequence of iid random variables each having characteristic function $\varphi_X(t)$. If $Y = \sum_{i=1}^{N} X_i$ then the characteristic function of Y is

A) $\varphi_X(P_N(t))$ B) $[\varphi_X(t)]^N$
C) $P_N(t)$ D) $P_N(\varphi_X(t))$

63. If $\{X_n\}$ is a branching process with $E[X_n]=\mu^n$ then the probability of ultimate extinction is 1 if

A) μ is finite B) $\mu \leq 1$

C) $\mu > 1$ D) None of these

64. In a Markov chain a recurrent state *i* is said to be null recurrent if and only if the mean recurrent time μ_i is

A) Zero B) One C) Infinity D) Negative

65. In time series analysis simple average method is used to calculate

A) Trend values B) Seasonal indices

C) Cyclic variations D) All the above

66. Laspeyre's index formula uses the weights of the

A) Current year

B) Base year

C) Average of the weights over a number of years

D) None of the above

67. Let Y_1, Y_2, Y_3 be three uncorrelated random variables with $E[Y_1] = \theta_1 + \theta_3$, $E[Y_2] = \theta_1 + \theta_2$ and $E[Y_3] = \theta_2 - \theta_3$, then which of the following is/are correct?

(a) $\theta_1 + \theta_2 + \theta_3$ is estimable.

(b) $\theta_1 + 2\theta_3$ is estimable.

A) Both (a) and (b) B) Neither(a) and (b)

C) (a) alone D) (b) alone

68. Which of the following parametric functions is a contrast?

A) $\theta_1 + 2\theta_2 - \theta_3$ B) $2\theta_1 - \theta_2 + 2\theta_3$ C) $2\theta_1 - 3\theta_2 + \theta_3$ D) $\theta_1 + \theta_2 - \theta_3$

69. A block design is said to be connected if

A) All elementary linear contrasts are estimable

B) Number of blocks is same as number of treatments

C) Number of blocks is greater than number of treatments

D) None of the above

70. Which of the following designs does not apply all the three basic principles of design of experiments?

A) RBD B) CRD C) LSD D) GLSD

71. Error degrees of freedom for a Graeco Latin square design of size 5 is

A) 8 B) 24 C) 4 D) 12

72. Error degrees of freedom corresponding to a 2^3 factorial experiment treplicated in r blocks is

A) 8r-1 B) 8(r-1) C) 7(r-1) D) 7r-1

73. For a 3^3 factorial experiment, if two effects are totally confounded with the blocks, then the block size reduces to

A) 3 B) 9 C) 6 D) 18

74. Let v be the number of treatments for a BIBD with b blocks each with k plots and repeats λ times in pair, then

A) $\lambda(r-1) = b(k-1)$ B) $\lambda(v-1) = r(k-1)$

C) $\lambda(b-1) = r(k-1)$ D) $b(v-1) = \lambda(k-1)$

75. In stratified random sampling, for a fixed sample size, the proportional allocation will minimize the variance of the unbiased estimator of the population mean if

A) the strata sizes are equal   B) the strata variances are equal

C) the strata means are equal   D) the strata means are unequal

76. The systematic sampling is superior to SRS when intraclass correlation become

A) Positive   B) Negative   C) 0   D) systematic sampling is always better than SRS

77. Consider the following statements on ratio and regression estimates of population mean $\bar{Y}$

(a) The ratio estimator of $\bar{Y}$ is biased.

(b) The regression estimate is generally more efficient than the ratio estimator.

A) Both (a) and (b)   B) Neither (a) nor (b)

C) (a) alone   D) (b) alone

78. The efficiency of SRSWOR with respect to SRSWR is

A) $\frac{N-n}{N-1}$ B) $\frac{n-1}{N-1}$ C) $\frac{N-1}{n-1}$ D) $\frac{N-1}{N-n}$

79. Which of the following relationship of sample size with sampling and non-sampling error is correct?

A) Both errors increase with sample size

B) Both errors decrease with sample size

C) Sampling error increases and non-sampling error decreases with sample size

D) Sampling error decreases and non-sampling error increases with sample size

80. Let C_x and C_y be coefficient of variations for the auxiliary variable and study variable and ρ is the correlation coefficient. Then ratio estimation is more efficient than sample mean when

A) $\rho\frac{C_y}{C_x} > \frac{1}{2}$ B) $\rho\frac{C_x}{C_y} < \frac{1}{2}$ C) $\rho\frac{C_y}{C_x} < \frac{1}{2}$ D) $\rho\frac{C_x}{C_y} > \frac{1}{2}$